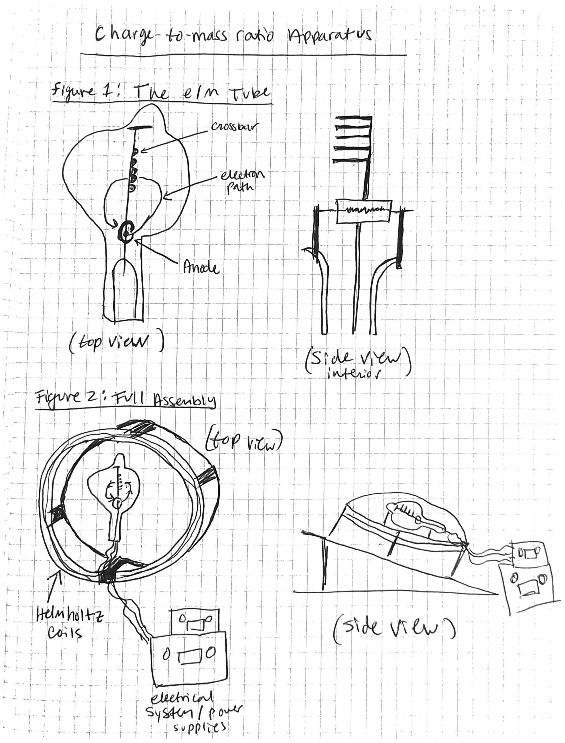
PHYS 2150
#5: Charge-to-Mass Ratio of the Electron

Cassidy Bliss &
Lab Partner: Sean Gopalakrishnan

Abstract/Introduction

In this lab, we measured the charge-to-mass ratio of an electron by accelerating electrons through a known change in potential by observing their paths within a uniform magnetic field. The apparatus used to accomplish this (described in further detail below) creates an environment where the electron moves in a magnetic field in a direction at right angles to the field. This means that the force on the electrons can be described within this apparatus as having magnitude: F = evB. This centripetal force remains perpendicular to the magnetic field, which in combination with the design of the apparatus having an anode with potential, moves the electron stream in a circular path completely perpendicular to the magnetic field. The radius of this circle is determined by the magnetic force/magnitude of the magnetic field. This in combination with relating kinetic energy of accelerating electrons through a potential difference, allows us to make the relation of mass to charge and eliminate the need to know the velocity of the electrons as such: $\frac{e}{m} = \frac{2V}{B^2 r^2}$. All data is then collected by observing these paths, reading current and voltage values, and calculating the magnetic field, set up by Helmholtz coils. The results were as follows: experimental value for charge-to-mass ratio ranged from 4.5 x 10^{11} to 5.5 x 10^{11} C/Kg, with an average of 1.5 x 10^{11} C/kG. The random uncertainty within the experiment was calculated to be around 3.22 x 10^9 C/Kg, and systematic uncertainty to be around 3.3 x 10^8 C/Kg. Total uncertainty was calculated to be 1.05 x 10^{19} C/Kg. Compared to the accepted value for charge to mass ratio of 1.759 x 10^{11} C/Kg, our value found to be off by about 2.52 x 10^{10} C/Kg. This error falls within our total calculated uncertainty, and therefore we can say that our value for the charge to mass ratio agrees with the accepted value. Possible sources of error were evaluated to the difference between the radii of the crossbars and the actual observed distance of the electron stream to the crossbar.

Apparatus:



Above is the apparatus used during this experiment. In figure 1, the e/m tube elements are shown. This is where the voltage is generated within the anode, and the the crossbars indicate the varying radii for the circular path of the electrons. The filament within the center of the anode provided the current of electrons. The side view of the cross bars and anode elements are also shown in figure 1. Figure 2 shows the full assembly of the e/m tube within the Helmholtz coils, where the magnetic field is generated. Wires stem from the e/m tube and the Helmholtz coils to provide the voltage, current, and also get output data required to specify the magnitude of the magnetic field. The path taken by the electrons for a particular radii is shown above within the e/m tube, which is in a plane perpendicular to the magnetic field generated by the coils. Resistors were also integrated into the electrical system. The e/m tube and Helmholtz coil assembly was secured on a platform that could be raised or lowed on one side to adjust the angle. This was required to cancel out the Earth's magnetic field, which may negatively effect the accuracy of the data, and was part of the calibration process (detailed below). We also used a compass for calibration and black fabric to block out ambient light.

Description of Experimental Procedure

Calibration for Earth's Magnetic Field

We first rearranged the axis of the Helmholtz coil in order to cancel the Earth's magnetic field. This was done by rotating the whole assembly to point in the direction of a compass needle. Then we adjusted the angle of the ramp which the assembly laid on, using a dip needle compass that could be rotated, and secured the angle where the dip needle pointed straight down.

Next, we turn on the electrical system in this order: anode power supply on and to 40 volts (accelerating potential to the tube), filament power supply on with voltage set to zero. We then slowly increased the voltage on the filament power supply until we could see the stream of electrons within the tube appear. As voltage increased, so did the current. We found 4 amps to be around the best for seeing the stream without exceeding 4.5 A (limit on current). We then turned on the power supply for the Helmholtz current, which provided the magnetic field. We adjusted the value of the current here until the electron beam appeared to be straight out to the side. This was carefully done, with setting at 40 volts for the anode. We recorded the value of the current from the Helmholtz power supply for an observed straight stream of electrons three separate times as I_0.

Obtaining Current for Varying Radii

With the anode set at 40 volts, we then began to increase the current which increased the magnetic field strength, until the stream became a circular path. We adjusted this current until the circle was nearly hitting the most outside crossbar (largest radii indicator), and recorded this value. We repeated this process for the remaining 4 crossbars and recorded each value. Then we adjusted the anode to 60 volts, and repeated the collection of current required to create the circular path for the 5 radii. We then adjusted the anode to 80 volts, and repeated the same process for each radii, and recording of current.

Magnetic Field Data Collection

The radius of the Helmholtz coils was acquired by measuring the outside diameter and the inside diameter with a meter stick. Because the thickness is not negligible, the average of the inside and the outside diameter will act as the diameter. We then shut down the electrical system in the opposite order which it was turned on: first the Helmholtz coil power supply down to 0 A and then off, then the filament voltage down to zero and power supply off, and last the anode voltage to 0 and then off.

Results

Data Input

Below is the collection of data from the experiment, including I_0 values, uncertainty in I, current measurements for each 5 crossbeams for each voltage settings, 40, 60 and 80. Magnetic field data is also included here.

In[104]:=
```
(*Measured I0 current in Amperes*)
I1 = 0.086;
I2 = 0.071;
I3 = 0.073;

(*measured uncertainty in I & V *)
δI = 0.005; (*amps*)
δV = 0.005; (*Volts*)
```

In[65]:=
```
(*Current measurement with Anode at 40 Volts for each radius in Amperes*)
IRad1V40 = 2.161;
IRad2V40 = 2.387;
IRad3V40 = 2.691;
IRad4V40 = 3.116;
IRad5V40 = 3.715;
(*Current measurement with Anode at 60 Volts for each radius in Amperes*)
IRad1V60 = 2.602;
IRad2V60 = 2.876;
IRad3V60 = 3.268;
IRad4V60 = 3.804;
IRad5V60 = 4.504;
(*Current measurement with Anode at 80 Volts for each radius in Amperes*)
IRad1V80 = 2.966;
IRad2V80 = 3.290;
IRad3V80 = 3.725;
IRad4V80 = 4.353;
IRad5V80 = 5.207;

(*Known radii for each crossbar in meters*)
radius1 = (11.46 * 0.01) / 2 ;
radius2 = (10.27 * 0.01) / 2;
radius3 = (9.12 * 0.01) / 2;
radius4 = (7.69 * 0.01) / 2;
radius5 = (6.39 * 0.01) / 2;

(*uncertainty in radius of crossbars in meters*)
δrcrossbar = 0.17 * 0.01;

(*Magnetic Field Data*)
turns = 72;
outerDiameter = 67.6 * 0.01; (*m*)
innerDiameter = 64.4 * 0.01; (*m*)
avgDiameter = (((67.6 + 64.4) / 2) * 0.01); (*m*)
avgRadiusCB = avgDiameter / 2; (*m*)

(*uncertainty in radius of coils*)
δr = 0.01 * 0.01 (*m*)
```

Out[91]= 0.0001

Calculation of I_{net} :

I_{net} is calculated by selecting a representative I_0 and subtracting that value from each I_{total} current for each radii, and will be redefined as such here: $I_{net} = I_{total} - I_0$

In[32]:=
```
(*Inet with Anode at 40 Volts for each radius in Amperes*)
InetRad1V40 = IRad1V40 - I3 ;
InetRad2V40 = IRad2V40 - I3;
InetRad3V40 = IRad3V40 - I3;
InetRad4V40 = IRad4V40 - I3;
InetRad5V40 = IRad5V40 - I3;
(*Inet with Anode at 60 Volts for each radius in Amperes*)
InetRad1V60 = IRad1V60 - I3;
InetRad2V60 = IRad2V60 - I3;
InetRad3V60 = IRad3V60 - I3;
InetRad4V60 = IRad4V60 - I3;
InetRad5V60 = IRad5V60 - I3;
(*Inet with Anode at 80 Volts for each radius in Amperes*)
InetRad1V80 = IRad1V80 - I3;
InetRad2V80 = IRad2V80 - I3;
InetRad3V80 = IRad3V80 - I3;
InetRad4V80 = IRad4V80 - I3;
InetRad5V80 = IRad5V80 - I3;
```

Uncertainty in Magnetic Field

In[101]:=
```
(*Uncertainty in Magnetic Field of representative data
 point using error propagation sna d fractional uncertainty*)
```
$$B = \frac{8 * 4 * \pi * 10^{-7} * turns * InetRad3V60}{\sqrt{125} * avgRadiusCB^2}$$

Out[101]= 0.00189942

In[103]:= $$\delta B = B * \sqrt{\left(\left(\frac{\delta I}{InetRad3V60}\right)^2 + \left(\frac{\delta r}{avgRadiusCB}\right)^4\right)}$$

Out[103]= 2.97248×10^{-6}

Computation of e/m (mass to charge ratio)

The ratio of charge to mass (e/m) is calculated as such: $\frac{e}{m} = \frac{2V}{B^2 r^2}$, where $B = \frac{8 u_0 NI}{\sqrt{125}\, a^2}$. This calculation is done below for each of the 15 measurements of current for each crossbar radius and each voltage setting.

In[47]:=

```
(*Mass to charge ratio for Radii 1-5 with 40 Volts in units of C/Kg*)
```

$$emRatioR1V40 = \frac{3.91 * avgRadiusCB^2 * 40}{(4 * \pi * 10^{-7})^2 * turns^2 * InetRad1V40^2 * radius1^2}$$

Out[47]= 1.45348×10^{11}

In[48]:= $$emRatioR2V40 = \frac{3.91 * avgRadiusCB^2 * 40}{(4 * \pi * 10^{-7})^2 * turns^2 * InetRad2V40^2 * radius2^2}$$

Out[48]= 1.47358×10^{11}

In[49]:= $$emRatioR3V40 = \frac{3.91 * avgRadiusCB^2 * 40}{(4 * \pi * 10^{-7})^2 * turns^2 * InetRad3V40^2 * radius3^2}$$

Out[49]= 1.45986×10^{11}

In[50]:= $$emRatioR4V40 = \frac{3.91 * avgRadiusCB^2 * 40}{(4 * \pi * 10^{-7})^2 * turns^2 * InetRad4V40^2 * radius4^2}$$

Out[50]= 1.51979×10^{11}

In[51]:= $$emRatioR5V40 = \frac{3.91 * avgRadiusCB^2 * 40}{(4 * \pi * 10^{-7})^2 * turns^2 * InetRad5V40^2 * radius5^2}$$

Out[51]= 1.53659×10^{11}

In[52]:=

```
(*Mass to charge ratio for Radii 1-5 for 60 Volts in units of C/Kg*)
```

$$emRatioR1V60 = \frac{3.91 * avgRadiusCB^2 * 60}{(4 * \pi * 10^{-7})^2 * turns^2 * InetRad1V60^2 * radius1^2}$$

Out[52]= 1.48616×10^{11}

In[53]:= $$emRatioR2V60 = \frac{3.91 * avgRadiusCB^2 * 60}{(4 * \pi * 10^{-7})^2 * turns^2 * InetRad2V60^2 * radius2^2}$$

Out[53]= 1.50641×10^{11}

In[54]:= $$emRatioR3V60 = \frac{3.91 * avgRadiusCB^2 * 60}{(4 * \pi * 10^{-7})^2 * turns^2 * InetRad3V60^2 * radius3^2}$$

Out[54]= 1.47028×10^{11}

In[55]:= $$emRatioR4V60 = \frac{3.91 * avgRadiusCB^2 * 60}{(4 * \pi * 10^{-7})^2 * turns^2 * InetRad4V60^2 * radius4^2}$$

Out[55]= 1.51645×10^{11}

In[56]:= $$\text{emRatioR5V60} = \frac{3.91 * \text{avgRadiusCB}^2 * 60}{(4 * \pi * 10^{-7})^2 * \text{turns}^2 * \text{InetRad5V60}^2 * \text{radius5}^2}$$

Out[56]= 1.55713×10^{11}

In[57]:= (*Mass to charge ratio for Radii 1-5 for 80 Volts in units of C/Kg *)

$$\text{emRatioR1V80} = \frac{3.91 * \text{avgRadiusCB}^2 * 80}{(4 * \pi * 10^{-7})^2 * \text{turns}^2 * \text{InetRad1V80}^2 * \text{radius1}^2}$$

Out[57]= 1.51427×10^{11}

In[58]:= $$\text{emRatioR2V80} = \frac{3.91 * \text{avgRadiusCB}^2 * 80}{(4 * \pi * 10^{-7})^2 * \text{turns}^2 * \text{InetRad2V80}^2 * \text{radius2}^2}$$

Out[58]= 1.52485×10^{11}

In[59]:= $$\text{emRatioR3V80} = \frac{3.91 * \text{avgRadiusCB}^2 * 80}{(4 * \pi * 10^{-7})^2 * \text{turns}^2 * \text{InetRad3V80}^2 * \text{radius3}^2}$$

Out[59]= 1.50044×10^{11}

In[60]:= $$\text{emRatioR4V80} = \frac{3.91 * \text{avgRadiusCB}^2 * 80}{(4 * \pi * 10^{-7})^2 * \text{turns}^2 * \text{InetRad4V80}^2 * \text{radius4}^2}$$

Out[60]= 1.53649×10^{11}

In[61]:= $$\text{emRatioR5V80} = \frac{3.91 * \text{avgRadiusCB}^2 * 80}{(4 * \pi * 10^{-7})^2 * \text{turns}^2 * \text{InetRad5V80}^2 * \text{radius5}^2}$$

Out[61]= 1.54652×10^{11}

In[131]:=
```
(*Average Value of e/m*)
AvgEMRatio =
 (emRatioR1V40 + emRatioR2V40 + emRatioR3V40 + emRatioR4V40 + emRatioR5V40 + emRatioR1V60 +
    emRatioR2V60 + emRatioR3V60 + emRatioR4V60 + emRatioR5V60 + emRatioR1V80 +
    emRatioR2V80 + emRatioR3V80 + emRatioR4V80 + emRatioR5V80) / 15
```

Out[131]= 1.50682×10^{11}

```
(*List of em Ratio values*)
Thread[{emRatioR1V40, emRatioR2V40, emRatioR3V40, emRatioR4V40, emRatioR5V40,
  emRatioR1V60, emRatioR2V60, emRatioR3V60, emRatioR4V60, emRatioR5V60,
  emRatioR1V80, emRatioR2V80, emRatioR3V80, emRatioR4V80, emRatioR5V80}]
```

Out[ ]= $\{1.45348 \times 10^{11}, 1.47358 \times 10^{11}, 1.45986 \times 10^{11}, 1.51979 \times 10^{11}, 1.53659 \times 10^{11},$
$1.48616 \times 10^{11}, 1.50641 \times 10^{11}, 1.47028 \times 10^{11}, 1.51645 \times 10^{11}, 1.55713 \times 10^{11},$
$1.51427 \times 10^{11}, 1.52485 \times 10^{11}, 1.50044 \times 10^{11}, 1.53649 \times 10^{11}, 1.54652 \times 10^{11}\}$

Error and Uncertainty Analysis of E/M Ratio

In[127]:=
```
(*Accepted value for e/m*)
emRatioActual = 1.759 * 10^11;

(*Standard deviation of theoretical e/m / Random Error*)
```

$$\sigma EMRatio = \sqrt{\left(\frac{1}{15-1} * \left((emRatioR1V40 - AvgEMRatio)^2 + (emRatioR2V40 - AvgEMRatio)^2 + (emRatioR3V40 - AvgEMRatio)^2 + (emRatioR4V40 - AvgEMRatio)^2 + (emRatioR5V40 - AvgEMRatio)^2 + (emRatioR1V60 - AvgEMRatio)^2 + (emRatioR2V60 - AvgEMRatio)^2 + (emRatioR3V60 - AvgEMRatio)^2 + (emRatioR4V60 - AvgEMRatio)^2 + (emRatioR5V60 - AvgEMRatio)^2 + (emRatioR1V80 - AvgEMRatio)^2 + (emRatioR2V80 - AvgEMRatio)^2 + (emRatioR3V80 - AvgEMRatio)^2 + (emRatioR4V80 - AvgEMRatio)^2 + (emRatioR5V80 - AvgEMRatio)^2\right)\right)}$$

Out[136]= 3.21543×10^9

In[143]:=
```
(*Uncertainty in EM Ratio*)

δRcrossBars = 0.07 * 0.01; (*meters*)

(*Sytematic
 Error: propagation of uncertainty through em using representative data point*)
```

$$\delta em = AvgEMRatio * \sqrt{\left(\frac{\delta B}{B}\right)^2 + \left(\frac{\delta r}{radius3}\right)^4 + \left(\frac{\delta I}{InetRad3V60}\right)^2}$$

Out[144]= 3.33485×10^8

In[150]:=
```
(*Error Analysis*)

(*error value*)
EMerror = (Abs[AvgEMRatio - emRatioActual])
```

In[153]:=
```
(*Total Uncertainty (sum of uncertainty and random error*)
totalUncertainty = σEMRatio^2 + δem^2
```

Out[153]= 1.04502×10^{19}

Discussion

In this lab, we used Helmholtz coils and an apparatus accelerating a stream of electrons in circles of varying radii in order to experimentally derive a value for the charge to mass ratio of the electron. Using three different voltage settings and 5 varying radii, we recorded the current values that completed the circle of our radii. Using the formula: $\frac{e}{m} = \frac{2V}{B^2 r^2}$, our experimental value for e/m ranged from around

1.45 *x* 10^{11} to 1.55 *x* 10^{11} . Random uncertainty was calculated to be around 3.22 x 10^{9}, using standard deviation. Systematic uncertainty was calculated to be around 3.3 x 10^{8}. The accepted value of the charge-to-mass ratio is 1.758 x 10^{11}. Our average experimental value of 1.5 x 10^{11} was off by around 2.52 x 10^{10}. Total uncertainty was calculated by adding the squares of our random and systematic uncertainty and was found to be 1.05 x 10^{19}. Therefore, we can conclude that our experimental value agrees with the accepted value, because it falls within our total uncertainty. There were numerous possible sources of error within this experiment that may have contributed to our overall error. When measuring the current for which the circle of electron stream was “at” the crossbar is the main source of error to consider for our experiment. It was difficult to see, given the shape and height of the apparatus, where exactly the stream of electrons were in relation to the crossbar. when closest to the crossbar, the electrons began hitting it and the circle could not be said to be complete. Therefore, we adjusted current until “nearly” hitting the crossbar. However, the uncertainty for how close we truly were to the crossbar may have been nearly 0.1 cm, as it was difficult to tell and looked different from varying angles. Much of our error may have come from this because the current responsible for the circle for the “expected” radii may not have corresponded to the radii used within the equation. Another source of error may have come from the I_0 values recorded when calibrating to the Earth’s magnetic field. Because of the shape and angles of the e/m tube, it was difficult to tell when the electron stream was exactly straight. This may have caused error propagating through our experimental calculations for the charge to mass ratio also.

Conclusion

In conclusion, we were able to experimentally derive a value for the charge to mass ratio (e/m) of the electron. our experimental value for e/m ranged from around 1.45 *x* 10^{11} to 1.55 *x* 10^{11} . Random uncertainty was calculated to be around 3.22 x 10^{9}, using standard deviation. Systematic uncertainty was calculated to be around 3.3 x 10^{8}. The accepted value of the charge-to-mass ratio is 1.758 x 10^{11}. Our average experimental value of 1.5 x 10^{11} was off by around 2.52 x 10^{10}. Total uncertainty was calculated by adding the squares of our random and systematic uncertainty and was found to be 1.05 x 10^{19}. Therefore, we can conclude that our experimental value agrees with the accepted value, because it falls within our total uncertainty.

Scanned Sheets from Lab Notebook

Charge-to-mass Ratio of the electron
PHYSICS 2150
Experiment #9
University of Colorado

10/19/2021
Cassidy Bliss
Lab partner: Sean Gopalakrishnan

Part 1: Determination of I_0

~~I_0 #1: Voltage = 0.01 ± 0.005~~
~~Amps = 0.01 ± 0.005~~ wasn't moving straight down

I_0 #2: Voltage = 0.021 ± 0.005 V
Amps = 0.086 ± 0.005 A

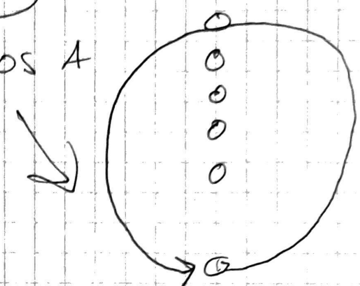
I_0 #3: Voltage 0.11 ± 0.005 V
Amps: 0.071 ± 0.005 A

I_0 #4: Votage: 0.17 ± 0.005 V
Amps: 0.073 A ± 0.005 A

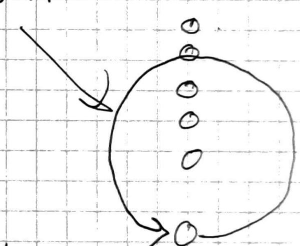
10/19
Cassidy
DUSP

1st circle : outermost bar [Anode set @ 40 volts]

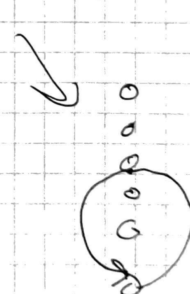
Bar 1: I_{total} = 2.161 A ± 0.005 A



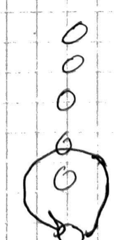
Bar #2: I_{total} = 2.387 ± 0.005 A



Bar #3: I_{total} = 2.691 ± 0.005 A

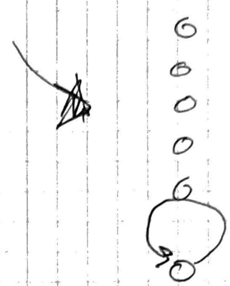


Bar #4: I_{total} = 3.116 ± 0.005 A



10/19
Cassidy Bliss

Bar #5: I_{total} = 3.715 ± 0.005 A



Anode set @ 60 V

Bar 1: I_{tot} = 2.602 ± 0.005 A

Bar 2: I_{tot} = 2.876 ± 0.005 A

Bar 3: I_{tot} = 3.268 ± 0.005 A

Bar 4: I_{tot} = 3.804 ± 0.005 A

Bar 5: I_{tot} = 4.504 ± 0.005 A

[Anode set at 80V]

Bar #1: I_{tot} = 2.966 ± 0.005 A

Bar #2: I_{tot} = 3.290 ± 0.005 A

Bar #3: I_{tot} = 3.725 ± 0.005 A

Bar #4: I_{tot} = 4.353 ± 0.005 A

Bar #5: I_{tot} = 5.207 ± 0.005 A

Magnetic Field data

turns (N) = 72

Diameter outer: 67.6 cm ± 0.1 cm

Diameter inner: 64.4 cm ± 0.1 cm

Avg Diameter = $\frac{67.6 + 64.4}{2}$ ± 0.1 cm

Keith [signature]
10/19/21