PHYS 2150
#6: Mass of K0 Meson

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Abstract/Introduction

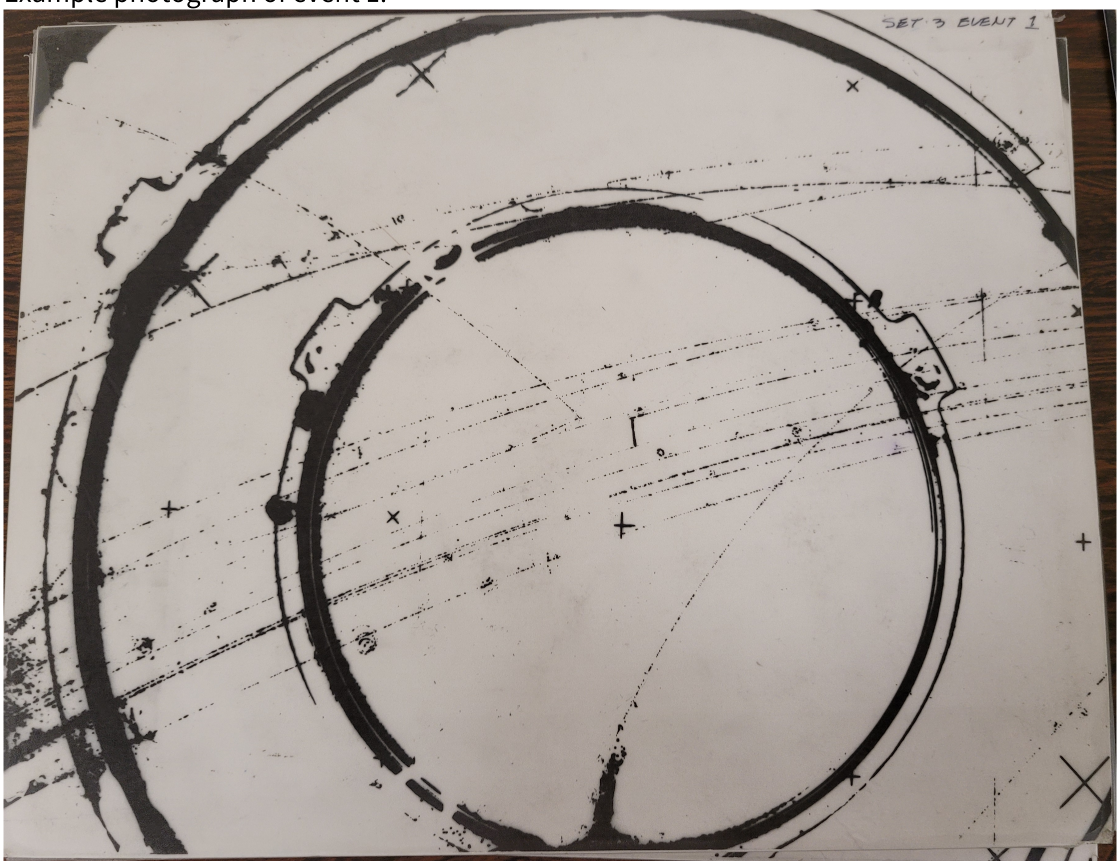
The purpose of this experiment is to determine the mass and the lifetime of the K0 meson. When examined inside of a bubble chamber, K+ mesons can be seen by the tracks they leave behind, along with the tracks of their decay products (pi + and pi- mesons). Using the laws of conservation of energy and momentum and data from the tracks these particles leave behind, we can determine the mass and lifetime of the invisible K0 particle (K0 meson). We did this for 5 events of K0 mesons photographed within a bubble chamber. Our results were not particularly satisfactory. We first experimentally derived the magnetic field strength, found to be the following for each event: Event 1: 1.38 T; Event 2: 1.32 T; Event 3: 1.34 T; Event 4: 1.34 T; Event 5: 1.34 T. Our reported and therefore calculated uncertainty of 0.01 T was far too low and this resulted in a disagreement between experimental and accepted value of 1.4 T. In part 2, we experimentally derived the rest mass of the K^0 meson. The results were as follows for each event : Event 1: 439.0 MeV; Event 2: 558.5 MeV; Event 3: 866.99 MeV; Event 4: 426.7 MeV ; Event 5: 792.5 MeV. Our uncertainty was again too low for our results to be said to agree with the accepted value of 497.67 MeV. In part 3 we experimentally derived the lifetime of the K^0 meson. The average lifetime was calculated to be 7.97437×10^{-11} seconds compared to the accepted value of 8.9×10^{-11} seconds. Our average results agreed with the accepted value given our calculated uncertainty of 0.35 seconds.

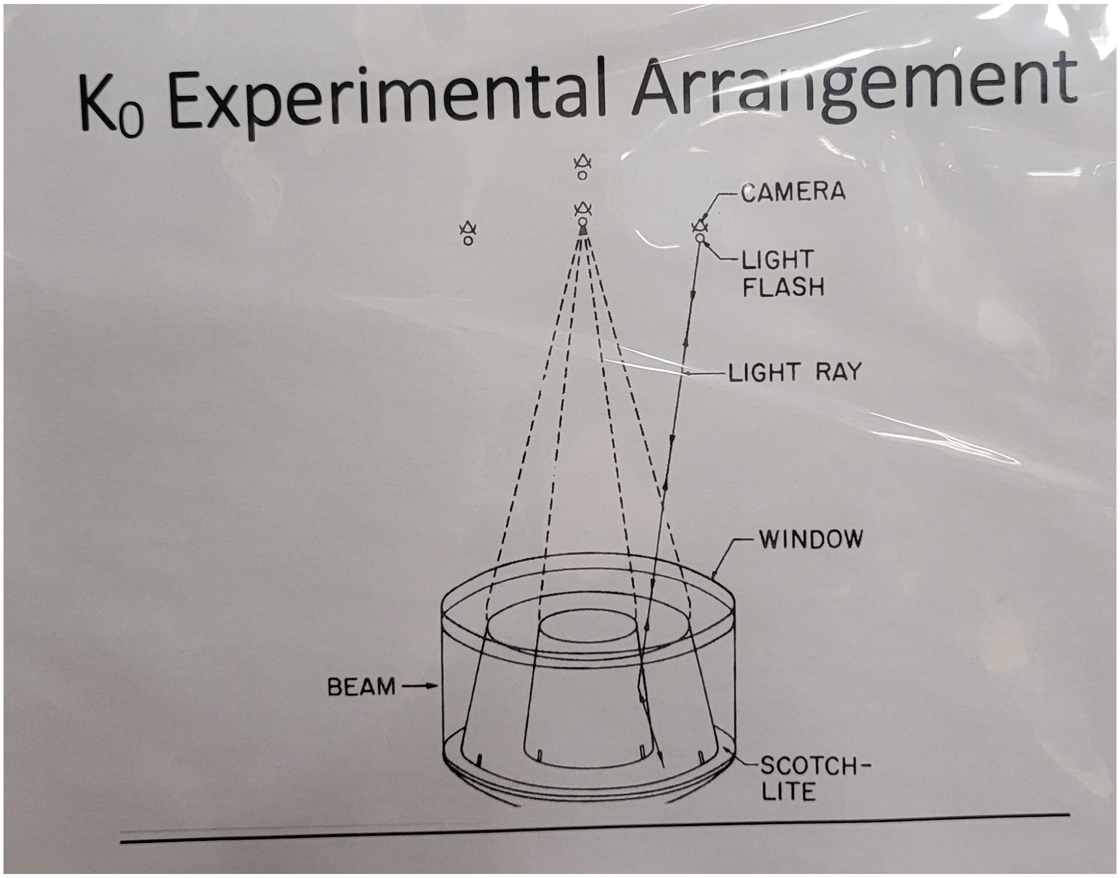
Apparatus/Materials:

In this experiment, we did not use a bubble chamber directly to obtain data of the K+ mesons and their decay products. Instead, we were given photographs from 5 of these events within a bubble chamber. However, in order to derive mass and lifetime of a K0 meson, it is important to understand how it works. Materials also used in this lab were a standard ruler, a protractor, tracing paper, and a transparent arc radius template.

Within a bubble chamber is a mixture of organic liquids. As the K0 mesons enter the chamber, one or more may interact with the neutrons inside of the molecules of the liquid, causing the decay reaction. When a K+ (visible) interacts with a neutron in this way, the products are a proton(visible) and a K0 meson(invisible). The K0 meson then quickly decays into one positive pion and one negative pion(both visible). During these events, a magnetic field is applied to the bubble chamber, applying a centripetal force to the charged particles, and causing them to move in a circle. Pictures are taken from above to show the tracks that the charged particles left behind.

Example photograph of event 1:





Above is a diagram of the set up of a bubble chamber.

Description of Experimental Procedure

Part 1: Determinations of the Magnetic Field:

Using the photographs and the transparent template of known radii curves, we measured the radius of the curvature of a representative K+ track stretching across most of the photograph. We did this for each event 1-5. We recorded each radius relative to each event as well as the uncertainty. Using these values, along with an accepted value of momentum for K^+, the experimental value for the magnetic field can be obtained.

Part 2: Determination of the K0 mass:

For each event, we traced the incident K^+ track, the photon track, and the pion track curves. We then measured the radius of the curvature of the tracks the pions left behind (using the transparent radii template), the angles of each pion from the axis from which the K^{+} trajectory decays (using a protractor and ruler), the total angle between the two pions (using a protractor and ruler), and the length of the K^{0} track (with *a* ruler) by measuring from where the proton branches off to where the pions begin propagating from each other (at a point). We then measured the distance between two fiducial marks on the photograph with a ruler for scaling purposes later on. Each of these measurements along with uncertainties were recorded.

Part 3: Determination of the K0 lifetime

This part is all calculation based, and will be using values experimentally measured from both part 1 and part 2 in order to determine the lifetime of the K^{0} meson. more details of these calculations are included in the Results/calculation section of part 3 below.

Results

Part 1: Determination of the Magnetic Field

The first section below is the code storing data taken, uncertainties, and known values required for calculations.

Data input:

```
(*known values*)
actualTickLen = N[10 / 100]; (*m*)
tickMarkLenE1 = N[7.4 / 100]; (*m*)
tickMarkLenE2 = N[7 / 100]; (*m*)
tickMarkLenE3 = N[7.4 / 100]; (*m*)
tickMarkLenE4 = N[7.4 / 100]; (*m*)
tickMarkLenE5 = N[7.4 / 100];(*m*)
momentumK = 595; (*MeV/c*)
kCharge = N[1.6 * 10^-19]; (*coulombs*)
unitFactor = 2.998 * 10^2; (*MeV/cmT*)

actualMagField = 1.4; (*T*)

(*Scaling Factors for each event: picture to actual size*)
scaleE1 = N[actualTickLen / tickMarkLenE1]; (*m*)
scaleE2 = N[actualTickLen / tickMarkLenE2]; (*m*)
scaleE3 = N[actualTickLen / tickMarkLenE3]; (*m*)
scaleE4 = N[actualTickLen / tickMarkLenE4]; (*m*)
scaleE5 = N[actualTickLen / tickMarkLenE5]; (*m*)

(*uncertainties*)
δR = N[1 / 100]; (*m*) (*uncertainty of radius*)
δθ = 1; (*degree*)(*uncertainty of angle*)
δL = N[(0.1 / 100) * scaleE1]; (*m*)(*uncertainty of track length*)
δS = N[(0.1 / 100) * scaleE1]; (*m*)(*uncertainty of scale*)

(*Part 1 Data*)

(*Radius of each K+ Meson*)
kRadius1 = N[(115 / 100) * scaleE1]; (*m*)
kRadius2 = N[(105 / 100) * scaleE2]; (*m*)
kRadius3 = N[(110 / 100) * scaleE3]; (*m*)
kRadius4 = N[(110 / 100) * scaleE4]; (*m*)
kRadius5 = N[(110 / 100) * scaleE5]; (*m*)
```

Calculations and Results Part 1:

In this section, we use a known value for the momentum of K^+ meson, our measured raidus of K^+ and *a* unit factor to compute the experimental magnetic field strength.

The formula used for this part is: B = $p/(r *$ unitFactor) Results can be seen below in the output statements. These results were around 4 to 8 percent off from the actual value of 1.4 T. The uncertainty of the magnetic field B was found to be the same as the uncertainty in the radius (δB = δr = 0.01 T), because all other factors in the calculation are exact/accepted values. This was found to be far too small of an uncertainty given our results. It is easy to think back on the way we recorded our uncertainty using the template and conclude that we were far too confident in our measurements. We were at a minimum of around 4.6 percent off in some events, and in others off by around 9 percent. Because our uncertainty was unreasonable small, our results cannot be said to agree with the actual magnetic field strength. In the case we had recorded a more appropriate uncertainty, we probably could have said otherwise. A qualitative representation of the difference between measured and actual magnetic field is provided as well.

```
B1 = momentumK / (unitFactor * (kRadius1)) (*Experimental Magnetic Field 1 in T*)
Out[255]= 1.27708

B2 = momentumK / (unitFactor * (kRadius2)) (*Experimental Magnetic Field 2 in T*)
Out[254]= 1.3231

B3 = momentumK / (unitFactor * (kRadius3)) (*Experimental Magnetic Field 3 in T*)
Out[256]= 1.33513

B4 = momentumK / (unitFactor * (kRadius4)) (*Experimental Magnetic Field 4 in T*)
Out[257]= 1.33513

B5 = momentumK / (unitFactor * (kRadius5)) (*Experimental Magnetic Field 2 in T*)
Out[258]= 1.33513

(*Uncertainty of Magnetic Field in Teslas*)
δp = δR
Out[585]= 0.01

(*Error of Magnetic Field from Event 1*)
error1 = (Abs[B1 - actualMagField]) / actualMagField

0.08779766220958884`

In[359]:= percentError1 = (error1 * 100)
Out[359]= 8.77977
```

```
(*Error of Magnetic Field from Event 2*)
error2 = (Abs[B2 - actualMagField]) / actualMagField
```

Out[360]= 0.0549255

```
In[361]:= percentError2 = (error2 * 100)
```

Out[361]= 5.49255

```
(*Error of Magnetic Field from Event 3*)
error3 = (Abs[B3 - actualMagField]) / actualMagField
```

Out[362]= 0.0463339

```
In[363]:= percentError3 = (error3 * 100)
```

Out[363]= 4.63339

```
In[391]:= (*Error of Magnetic Field from Event 4*)
error4 = (Abs[B4 - actualMagField]) / actualMagField
```

Out[391]= 0.0463339

```
In[365]:= percentError4 = (error4 * 100)
```

Out[365]= 4.63339

```
In[392]:= (*Error of Magnetic Field from Event 5*)
error5 = (Abs[B5 - actualMagField]) / actualMagField
```

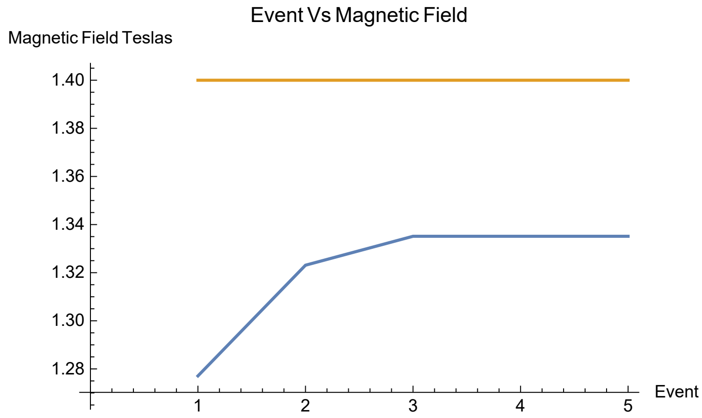
Out[392]= 0.0463339

```
In[367]:= percentError5 = (error5 * 100)
```

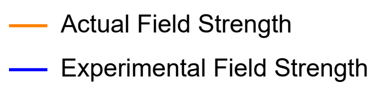
Out[367]= 4.63339

In[516]:=
```
(*Visual representation of Magnetic field found against actual*)
Show[%395, AxesLabel → {HoldForm[Event], HoldForm[Magnetic Field Teslas]},
 PlotLabel → HoldForm[Event Vs Magnetic Field], LabelStyle → {GrayLevel[0]}]
LineLegend[{Orange, Blue}, {"Actual Field Strength", "Experimental Field Strength"}]
```

Out[516]=



Out[517]=



Part 2: Determination of the K0 mass

The first section below is a collection of data taken including units on the right side. This includes the radius of each pion, the angles of each pion, the total angle between the pions, and the length of K^0 track. Each of these are included for each event, and scaled to real life (within the chamber in which the photos were taken)

Data input:

In[460]:=
```
(*Event 1*)
p1RadiusE1 = N[(55 / 100) * scaleE1]; (*m*)
p2RadiusE1 = N[(70 / 100) * scaleE1]; (*m*)


theta1E1 = 42 * scaleE1; (*degree*)
theta2E1 = 27 * scaleE1; (*degree*)
thetaTotE1 = 69 * scaleE1; (*degree*)

kTrackLenE1 = (1.6 / 100) * scaleE1; (*m*)

(*Event 2*)
p1RadiusE2 = (26 / 100) * scaleE2; (*m*)
p2RadiusE2 = (115 / 100) * scaleE2; (*m*)

theta1E2 = 59 * scaleE2; (*degree*)
theta2E2 = 30 * scaleE2; (*degree*)
```

```
thetaTotE2 = 89 * scaleE2; (*degree*)

kTrackLenE2 = (0.3 / 100) * scaleE2; (*m*)

(*Event 3*)
p1RadiusE3 = (85 / 100) * scaleE3; (*m*)
p2RadiusE3 = (75 / 100) * scaleE3; (*m*)

theta1E3 = 34 * scaleE3; (*degree*)
theta2E3 = 47 * scaleE3; (*degree*)
thetaTotE3 = 82 * scaleE3; (*degree*)

kTrackLenE3 = (1.1 / 100) * scaleE3; (*m*)

(*Event 4*)
p1RadiusE4 = (32 / 100) * scaleE4; (*m*)
p2RadiusE4 = (65 / 100) * scaleE4; (*m*)

theta1E4 = 35 * scaleE4; (*degree*)
theta2E4 = 45 * scaleE4; (*degree*)
thetaTotE4 = 80 * scaleE4; (*degree*)

kTrackLenE4 = (0.9 / 100) scaleE4; (*m*)

(*Event 5*)
p1RadiusE5 = (80 / 100) * scaleE5; (*m*)
p2RadiusE5 = (65 / 100) * scaleE5; (*m*)

theta1E5 = 36 * scaleE5; (*degree*)
theta2E5 = 46 * scaleE5; (*degree*)
thetaTotE5 = 82 * scaleE5; (*degree*)

kTrackLenE5 = (0 / 100) * scaleE5; (*m*)
```

Calculations and Results:

In this section, calculations are made to determine many things leading up to the final result of the rest mass of the K^0 meson. The first was calculating the momentum of each pion in MeV/c, using the formula: p = qrB/unitFactor. The next calculation was for the energy of each pion, which was found using the Pythagorean Relation of energy and momentum: E=$\sqrt{((mc^2)^2 + (pc)^2)}$ (for the sake of keeping units out of S.I., the c is not included). Total energy of $K^0$is then calculated using conservation of energy: E =$E_1 + E_2$, where the E1 and E2 are the energies of the pions 1 and 2, respectively. The momentum of the K^0 meson is then calculated using: p=$\sqrt{(p_1{}^2 + p_2{}^2 + 2\, p_1\, p_2 \cos(\theta))}$. Now we are able to solve for the rest mass of the K^0 meson using the Pythagorean relation once again where: mc^2 = $\sqrt{(E^2 - (pc)^2)}$ (again c is not used to keep units in MeV). Then the error was calculated, and found to be ranging from around 11 to 74 percent of the actual value. Event 1 was clearly the closest to the accepted value at 439.01 MeV. The uncertainty of the rest mass was calculated to be around 1.5 MeV using quadrature summation error propagation in tandem with the error in summation of perpendicular components of the momentum of K^0 (which should add up to be zero but did not). Our previously found systematic error within the measurement of the radius for each event propagating through these calculations, coupled with the fact that our uncertainty is rather low, resulted in our experimental results not agreeing with the accepted value. Given a more realistic uncertainty and more careful measurements of the radii of each pion, this may not have been the case. What may be the most interesting and discouraging results, is how random the calculated rest energies were in their positions away from the actual value. A visual representation of this is given below the calculated values for rest energy of K^0. In some events, it was found to be far too high, in others too low, and in others a little too high. There was not enough consistency to determine that the errors here were systematic.

```
In[529]:= (*Momentum of each Pion*)

(*known constants*)
unitFactorMom = 5.36 * 10^-22 ; (*kg*m/s- >>MeV/c unit conversion factor*)
restEpion = 139.57 ; (*rest engergy of pion in MeV*)
speedOfLight = 3 * 10^8 ; (*m/s*)
actualREK = 497.67; (*actual rest energy K^0 meson in MeV*)


momentump1E1 = kCharge * p1RadiusE1 *
  actualMagField / unitFactorMom (*momentum of pion1 Event 1 in MeV/c*)
Out[482]= 310.609

In[430]:= momentump2E1 = kCharge * p2RadiusE1 *
  actualMagField / unitFactorMom (*momentum of pion2 Event 1 in MeV/c*)
Out[430]= 395.321

In[468]:= momentump1E2 = kCharge * p1RadiusE2 *
  actualMagField / unitFactorMom (*momentum of pion1 Event 2 in MeV/c*)
Out[468]= 155.224

In[467]:= momentump2E2 = kCharge * p2RadiusE2 *
  actualMagField / unitFactorMom (*momentum of pion2 Event 2 in MeV/c*)
Out[467]= 686.567

In[466]:= momentump1E3 = kCharge * p1RadiusE3 *
  actualMagField / unitFactorMom (*momentum of pion1 Event 3 in MeV/c*)
Out[466]= 480.032

In[465]:= momentump2E3 = kCharge * p2RadiusE3 *
  actualMagField / unitFactorMom (*momentum of pion2 Event 3 in MeV/c*)
Out[465]= 423.558
```

In[473]:=
```
momentump1E4 = kCharge * p1RadiusE4 *
  actualMagField / unitFactorMom(*momentum of pion1 Event 4 in MeV/c*)
```
Out[473]= 180.718

In[474]:=
```
momentump2E4 = kCharge * p2RadiusE4 *
  actualMagField / unitFactorMom(*momentum of pion2 Event 4 in MeV/c*)
```
Out[474]= 367.084

In[475]:=
```
momentump1E5 = kCharge * p1RadiusE5 *
  actualMagField / unitFactorMom(*momentum of pion1 Event 5 in MeV/c*)
```
Out[475]= 451.795

In[476]:=
```
momentump2E5 = kCharge * p2RadiusE5 *
  actualMagField / unitFactorMom(*momentum of pion2 Event 5 in MeV/c*)
```
Out[476]= 367.084

(*For Energy of Each Pion:*)

(*energy of pion 1 of event 1 in MeV*)
energyp1E1 = $\sqrt{(\text{restEpion})^2 + (\text{momentump1E1})^2}$

Out[494]= 340.526

In[495]:= (*energy of pion 2 of event 1 in MeV*)
energyp2E1 = $\sqrt{(\text{restEpion})^2 + (\text{momentump2E1})^2}$

Out[495]= 419.235

In[496]:= (*energy of pion 1 of event 2 in MeV*)
energyp1E2 = $\sqrt{(\text{restEpion})^2 + (\text{momentump1E2})^2}$

Out[496]= 208.744

In[497]:= (*energy of pion 2 of event 2 in MeV*)
energyp2E2 = $\sqrt{(\text{restEpion})^2 + (\text{momentump2E2})^2}$

Out[497]= 700.61

In[498]:= (*energy of pion 1 of event 3 in MeV*)
energyp1E3 = $\sqrt{(\text{restEpion})^2 + (\text{momentump1E3})^2}$

Out[498]= 499.911

In[499]:= (*energy of pion 2 of event 3 in MeV*)
energyp2E3 = $\sqrt{(\text{restEpion})^2 + (\text{momentump2E3})^2}$

Out[499]= 445.961

In[500]:=
```
(*energy of pion 1 of event 4 in MeV*)
energyp1E4 = √((restEpion)^2 + (momentump1E4)^2)
```
Out[500]= 228.339

In[501]:=
```
(*energy of pion 2 of event 4 in MeV*)
energyp2E4 = √((restEpion)^2 + (momentump2E4)^2)
```
Out[501]= 392.721

In[502]:=
```
(*energy of pion 1 of event 5 in MeV*)
energyp1E5 = √((restEpion)^2 + (momentump1E5)^2)
```
Out[502]= 472.862

In[503]:=
```
(*energy of pion 2 of event 5 in MeV*)
energyp2E5 = √((restEpion)^2 + (momentump2E5)^2)
```
Out[503]= 392.721

```
(*For Energy K^0:*)
```

```
(*Energy of K^0 Event 1* in MeV)
  energyKE1= energyp1E1 + energyp2E1
```
Out[505]= 759.761

```
(*Energy of K^0 Event 2 in MeV*)
energyKE2 = energyp1E2 + energyp2E2
```
Out[506]= 909.354

```
(*Energy of K^0 Event 3 in MeV*)
energyKE3 = energyp1E3 + energyp2E3
```
Out[507]= 945.872

In[508]:=
```
(*Energy of K^0 Event 4 in MeV*)
energyKE4 = energyp1E4 + energyp2E4
```
Out[508]= 621.061

In[509]:=
```
(*Energy of K^0 Event 5 in MeV*)
energyKE5 = energyp1E5 + energyp2E5
```
Out[509]= 865.583

```
(*For momentum of K^0 meson:*)
```

```
(*K^0 momentum Event 1 in MeV/c*)
momentumKE1 =
 √ (momentump1E1^2 + momentump2E1^2 + (2 * momentump1E1 * momentump2E1 * Cos[thetaTotE1]))
```

Out[455]= 620.085

```
(*K^0 momentum Event 2 in MeV/c*)
momentumKE2 =
 √ (momentump1E2^2 + momentump2E2^2 + (2 * momentump1E2 * momentump2E2 * Cos[thetaTotE2]))
```

Out[463]= 717.618

```
(*K^0 momentum Event 3 in MeV/c*)
momentumKE3 =
 √ (momentump1E3^2 + momentump2E3^2 + (2 * momentump1E3 * momentump2E3 * Cos[thetaTotE3]))
```

Out[469]= 378.162

```
(*K^0 momentum Event 4 in MeV/c*)
momentumKE4 =
 √ (momentump1E4^2 + momentump2E4^2 + (2 * momentump1E4 * momentump2E4 * Cos[thetaTotE4]))
```

Out[472]= 451.3

```
(*K^0 momentum Event 5 in MeV/c*)
momentumKE5 =
 √ (momentump1E5^2 + momentump2E5^2 + (2 * momentump1E5 * momentump2E5 * Cos[thetaTotE5]))
```

Out[477]= 348.171

```
(*For Rest Energy of K^0 meson:*)

(*Rest energy of K^0 Event 1 in MeV:*)
restEnKE1 = √(energyKE1^2 - momentumKE1^2)
```

Out[511]= 439.012

```
(*Rest energy of K^0 Event 2 in MeV:*)
restEnKE2 = √(energyKE2^2 - momentumKE2^2)
```

Out[512]= 558.525

```
(*Rest energy of K^0 Event 3 in MeV:*)
restEnKE3 = √(energyKE3^2 - momentumKE3^2)
```

Out[513]= 866.987

```
(*Rest energy of K^0 Event 4 in MeV:*)
restEnKE4 = √(energyKE4^2 - momentumKE4^2)
```

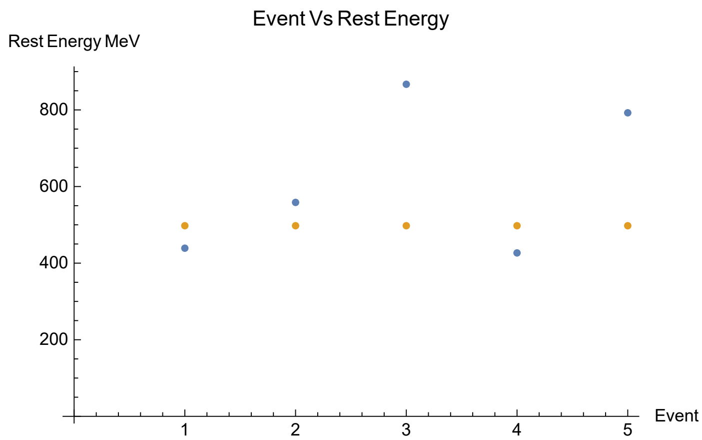
Out[514]= 426.666

```
(*Rest energy of K⁰ Event 5 in MeV:*)
restEnKE5 = Sqrt[energyKE5^2 - momentumKE5^2]
```

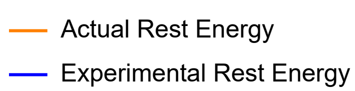
Out[515]= 792.472

In[602]:=
```
Show[%600, AxesLabel → {HoldForm[Event], HoldForm[Rest Energy MeV]},
 PlotLabel → HoldForm[Event Vs Rest Energy], LabelStyle → {GrayLevel[0]}]
LineLegend[{Orange, Blue}, {"Actual Rest Energy", "Experimental Rest Energy"}]
```

Out[602]=



Out[603]=



In[571]:=
```
(*Error in Rest Energy*)

errorREKE1 = Abs[actualREK - restEnKE1] / actualREK
```

Out[571]= 0.117865

In[573]:= `percentErrorREKE1 = errorREKE1 * 100`

Out[573]= 11.7865

In[574]:= `errorREKE2 = Abs[actualREK - restEnKE2] / actualREK`

Out[574]= 0.12228

In[575]:= `percentErrorREKE2 = errorREKE2 * 100`

Out[575]= 12.228

In[576]:= `errorREKE3 = Abs[actualREK - restEnKE3] / actualREK`

Out[576]= 0.742092

In[577]:= `percentErrorREKE3 = errorREKE3 * 100`

Out[577]= 74.2092

In[578]:=
```
errorREKE4 = Abs[actualREK - restEnKE4] / actualREK
```
Out[578]= 0.142672

In[579]:=
```
percentErrorREKE4 = errorREKE4 * 100
```
Out[579]= 14.2672

In[580]:=
```
errorREKE5 = Abs[actualREK - restEnKE5] / actualREK
```
Out[580]= 0.592364

In[581]:=
```
percentErrorREKE5 = errorREKE5 * 100
```
Out[581]= 59.2364

In[522]:=
```
(*Perpendicular Momentum*)

sumPerpMomE1 = (momentump1E1 * Sin[theta1E1]) + (momentump2E1 * Sin[theta2E1])
```
Out[522]= -306.065

In[523]:=
```
sumPerpMomE2 = (momentump1E2 * Sin[theta1E2]) + (momentump2E2 * Sin[theta2E2])
```
Out[523]= -540.068

In[524]:=
```
sumPerpMomE3 = (momentump1E3 * Sin[theta1E3]) + (momentump2E3 * Sin[theta2E3])
```
Out[524]= 710.341

In[525]:=
```
sumPerpMomE4 = (momentump1E4 * Sin[theta1E4]) + (momentump2E4 * Sin[theta2E4])
```
Out[525]= -361.681

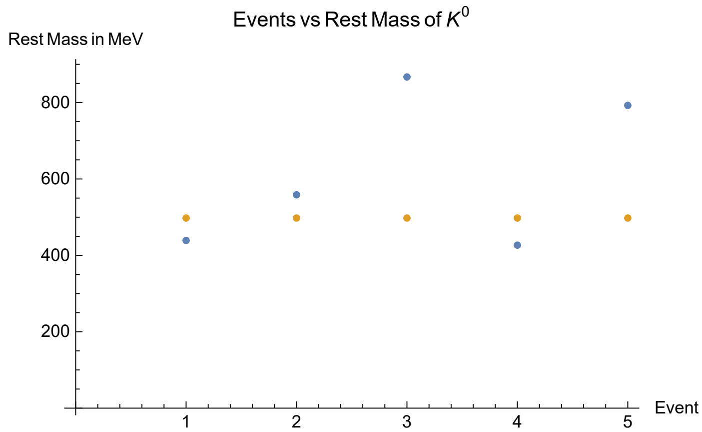
In[526]:=
```
sumPerpMomE5 = (momentump1E5 * Sin[theta1E5]) + (momentump2E5 * Sin[theta2E5])
```
Out[526]= -679.181

In[611]:=
```
(*uncertainty of Rest Mass K^0 meson*)
δRMTotal = √((sumPerpMomE1 / (2 * momentumKE1))^2 +
    (sumPerpMomE2 / (2 * momentumKE2))^2 + (sumPerpMomE3 / (2 * momentumKE3))^2 +
    (sumPerpMomE4 / (2 * momentumKE4))^2 + (sumPerpMomE5 / (2 * momentumKE5))^2)
```
Out[611]= 1.48206

```
(*Visual Representation of Actual Rest Mass to Measured Rest Mass*)
```

In[536]:=
```
Show[%533, AxesLabel → {HoldForm[Event], HoldForm[Rest Mass in MeV]},
 PlotLabel → HoldForm[Events vs Rest Mass of K⁰], LabelStyle → {GrayLevel[0]}]
LineLegend[{Orange, Blue}, {"Actual Rest Mass", "Measured Rest Mass"}]
```

Out[536]=



Out[537]=

Actual Rest Mass
Measured Rest Mass

Part 3: Determination of the K0 Lifetime

In this section, we calculate the experimental lifetime of the K^0 meson. We do this first by finding velocity using: $v = \frac{pc}{E}$. Since our momentum is in units of MeV/c, this formula gives us meters per second. We use the time dilation formula next, solving for gamma in each event, to find the lifetime of the K^0 meson from its own reference frame. Here: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ and $t_{proper=\gamma(t_{labFrame})}$. The uncertainty of the lifetime is calculated by propagating the uncertainty of both the momentum of K^0 and the track Length using quadrature summation. This uncertainty is found to be around 0.352 seconds. The average of the lifetimes given each event is then calculated and found to be: 7.97437×10^{-11}. Compared to the accepted value of $8.9 * 10^{-11}$, this was found to be about a 0.1 error or 10.4% error. Because the uncertainty was large enough in this case to cover our average error of 0.01 seconds, we can say that our results agree with the accepted value. This only happened however, because the results were averaged together. There was truly only one event in which the experimental lifetime was found to be considerably close to the accepted value (Event 4). The rest of the events were much further from the actual than the uncertainty, and it was essentially luck of widely dispersed results that led to an average within the uncertainty. A visual representation is given below the calculated results to show this more clearly.

```
(*For velocity of K0 Meson in m/s:*)

vKE1 = momentumKE1 * speedOfLight / energyKE1
Out[539]= 2.44847 × 10^8

In[540]:= vKE2 = momentumKE2 * speedOfLight / energyKE2
Out[540]= 2.36745 × 10^8

In[541]:= vKE3 = momentumKE3 * speedOfLight / energyKE3
Out[541]= 1.19941 × 10^8

In[542]:= vKE4 = momentumKE4 * speedOfLight / energyKE4
Out[542]= 2.17998 × 10^8

In[543]:= vKE5 = momentumKE5 * speedOfLight / energyKE5
Out[543]= 1.20672 × 10^8
```

```
In[555]:= (*k0 Flight Time correcting for time dilation*)
```

$$\gamma E1 = \frac{1}{\sqrt{1 - \frac{vKE1^2}{speedOfLight^2}}}$$

Out[555]= 1.73062

In[556]:= $$\gamma E2 = \frac{1}{\sqrt{1 - \frac{vKE2^2}{speedOfLight^2}}}$$

Out[556]= 1.62814

In[557]:= $$\gamma E3 = \frac{1}{\sqrt{1 - \frac{vKE3^2}{speedOfLight^2}}}$$

Out[557]= 1.09099

In[558]:= $$\gamma E4 = \frac{1}{\sqrt{1 - \frac{vKE4^2}{speedOfLight^2}}}$$

Out[558]= 1.45561

In[559]:= $$\gamma E5 = \frac{1}{\sqrt{1 - \frac{vKE5^2}{speedOfLight^2}}}$$

Out[559]= 1.09226

In[565]:=
```
(*Lifetime of K⁰ Meson in Event 1 in seconds*)
lifeTimeKE1 = γE1 * (kTrackLenE1 / vKE1)
```

Out[565]= 1.52825×10^{-10}

In[566]:=
```
(*Flight Time of K⁰ Meson in Event 2 in seconds*)
lifeTimeKE2 = γE2 * (kTrackLenE2 / vKE2)
```

Out[566]= 2.94736×10^{-11}

In[567]:=
```
(*Flight Time of K⁰ Meson in Event 3 in seconds*)
lifeTimeKE3 = γE3 * (kTrackLenE3 / vKE3)
```

Out[567]= 1.35211×10^{-10}

In[589]:=
```
(*Flight Time of K⁰ Meson in Event 4 in seconds*)
lifeTimeKE4 = γE4 * (kTrackLenE4 / vKE4)
```

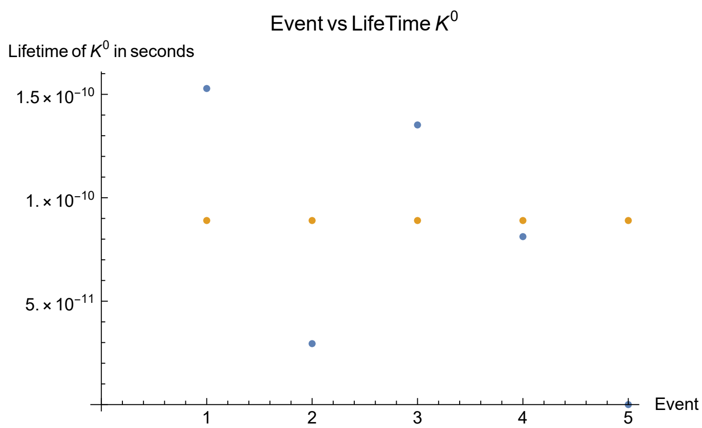
Out[589]= 8.12088×10^{-11}

In[569]:=
```
(*Flight Time of K⁰ Meson in Event 5 in seconds*)
lifeTimeKE5 = γE5 * (kTrackLenE5 / vKE5)
```

Out[569]= 0.

In[605]:=
```
Show[%604, AxesLabel → {HoldForm[Event], HoldForm[Lifetime of K⁰ in seconds]},
 PlotLabel → HoldForm[Event vs LifeTime K⁰], LabelStyle → {GrayLevel[0]}]
```

Out[605]=



```
(*Uncertainty in lifetime in seconds *)
```

In[612]:=
```
δLTKE = √((δL / kTrackLenE1)^2 + (δL / kTrackLenE2)^2 + (δL / kTrackLenE3)^2 +
    (δL / kTrackLenE4)^2 + (δp / momentumKE1)^2 + (δp / momentumKE2)^2 +
    (δp / momentumKE3)^2 + (δp / momentumKE4)^2 + (δp / momentumKE5)^2)
```
Out[612]= 0.352051

```
(*Average Lifetime Between Events in seconds*)

lifeTimeAvg = (lifeTimeKE1 + lifeTimeKE2 + lifeTimeKE3 + lifeTimeKE4 + lifeTimeKE5) / 5
```
Out[590]= 7.97437×10^{-11}

```
acceptedLifeTimeK = 8.9 * 10^-11; (*accepted lifetime of K^0meson in seconds*)
```
In[597]:=
```
errorLifeTime = Abs[(lifeTimeAvg - acceptedLifeTimeK)] / acceptedLifeTimeK
```
Out[597]= 0.104003

Discussion

In part 1, we used data from the radii of a representative K^{+} to determine *a* magnetic field strength for each event. The results were as follows:
Event 1: 1.38 T; Event 2: 1.32 T; Event 3: 1.34 T; Event 4: 1.34 T; Event 5: 1.34 T. The uncertainty of the magnetic field was calculated to be 0.01 T. The error between events from the accepted value of 1.4 T ranged from 4.63 percent to 8.78 percent. It was concluded that the uncertainty was reported as an unreasonably small value for the method used to determine the radii of the K^{+} tracks. In this method, some lines seemed nearly equally close to fitting the curve and therefore our uncertainty in hindsight should have been at the very least, half of the resolution of the template. We were overly confident and obviously did not measure carefully enough and therefore our results cannot be said to agree with the accepted value. If a more reasonable uncertainty were recorded for the radii, then our results very well may have agreed with the accepted value. Overall, I was satisfied with the results to some extent, but learned a hard lesson about uncertainty and measurement.

In part2, we collected data on the pions resulting from the K^{0}decay as well as the track length of the K^{0}meson by examining the visible photon track and start point of pion propagation. In the analysis, momentum of each pion and energy of each pion were calculated in order to find total energy and momentum of the incident K^{0} meson. This allowed us to derive *a* calculation for the rest energy of the K^{0}meson. The results were as follows: Event 1: 439.0 MeV; Event 2: 558.5 MeV; Event 3: 866.99 MeV; Event 4: 426.7 MeV ; Event 5: 792.5 MeV. The uncertainty was calculated from the uncertainty of momentum by summing the perpendicular components of momentum for each event and then propagating using quadrature summation. Comparing to the accepted value of 497.67 MeV ,the uncertainty is low compared to our error which ranged from 11.7 percent to 74.2 percent. Our results cannot be said to agree with the accepted value. With a larger uncertainty, the results may have agreed with the accepted value for Events 1,2, and 4. However, Event 3 and 5 were much too far off to agree with the accepted value under any reasonable condition of uncertainty. The method I reflect the most on that may have contributed to the error in this part goes back to using the template to measure the radii of the pions. With a more carefully traced trajectory of the photon and pion tracks, I believe that our angles, radii and track lengths would have been significantly more accurate. With more careful use of the template of known radii, the more accurate the overall rest energy would have been. Notably, Event 5 measured a 0.0 m track length, because it seemed to instantly decay into pions and there was not a visible length to measure. In this part, it seemed as though the lesson to learn was that uncertainty is not simply just representative of marks on a measurement tool, but also the quality of the picture and measurability of the object you are measuring, which should have been more carefully taken into account.

In part 3, measurements from part 1 and 2 and calculations done in these previous parts were used to calculate velocity and then flight time of the K^0 meson which represents the lifetime of the K^0 meson. Time dilation was required to correct for the time experienced in the K^0's reference frame due to its relativistic speed. The results for the experimental lifetimes were as follows: Event 1: 1.52825×10^{-10} sec; Event 2: 2.94736×10^{-11} sec; Event 3: 1.35211×10^{-10} sec ; Event 4: 8.12088×10^{-11} sec; Event 5: 0.0 sec. Our uncertainty for the lifetime was calculated to be 0.352051 second. Our lifetimes were averaged together, resulting in an average lifetime between events of 7.97437×10^{-11}. Compared to the accepted value of $8.9 \text{ x } 10^{-11}$, our average error was found to be 10.4 percent/ 0.104 seconds. Because our uncertainty is larger than our error, we can say in this case that our experimental value for the lifetime of the K^0 meson agrees with the accepted value. It should be noted however, that the average value of the lifetime may be deceiving and it was actually only in one event that we came substantially close to the accepted value (Event 4). Sources of error may have stemmed from measurement of the track length, which was traced onto paper and then measured. Using calipers could have given a more accurate reading than the ruler we used. Errors from momentum could have stemmed from a number of measurements, and therefore the same issues we ran into as described in part two could have easily played a part in our error. It does seem that random error played a part as well, given the inconsistency between positions from the accepted value. These errors would be more difficult to pin down and it would be helpful to redo this particular lab with refined methods, and more careful tracing, and radii matching before drawing any solid conclusions on the random errors.

Conclusion

In conclusion, we came within the ballpark of the accepted magnetic field strength values, however, because our uncertainty was unreasonably low given the conditions, we cannot say that our experimental values agree with the accepted value of 1.4 Teslas. In part 2, we found that none of our rest mass values for K^0 agreed with the accepted value of 497.67 MeV given the calculated uncertainty. The errors in our measurements and possibly tracing may have contributed to this propagation of errors found in rest mass. In part 3, we were able to say that our average experimental lifetime for the K^0 meson of 7.97437×10^{-11} seconds agreed with the accepted value of 8.9×10^{-11} seconds. Overall I would say that the error and uncertainty of parts 1 and 2 were troubling, and that obviously there were a series of measurements that were not done very well. This could be corrected both by tracing the events more carefully, and using more precise measurement devices. The uncertainty should have also been adjusted for most all measurements taken in this lab to reflect not just the resolution of the marks on the measurement device but also accounting for reliability of drawing and circumstantial uncertainty due to limitations of the measurements. While I was happy to see our result for the average lifetime in part 3 agreed with the accepted value, I think this was really depended on luck of averaging drastically wrong results from each event as noted before and therefore I would not agree that the experiment was a success overall. If given the time to do it again, I believe it could be done much better and results far more accurate

Scanned Sheets from Lab Notebook

Don’t forget to scan and attach the relevant pages from your lab notebook.

Mass of the K^0 meson
PHYS 2150
Experiment 6
University of Colorado

9/21/2021
Cassidy Bliss
Labpartner: Sean Gopalakrishnan

part 1

K^+ track raddi : ~~[illegible]~~ 115 (misread) cm ; picture 1 ± 1cm uncertainty
K^+ track radii : 105 cm ; picture 2 ± 1 cm uncertainty
K^+ track radii : 110 cm ; picture 3 ± 1cm uncertainty
K^+ track radii : 110 cm ; picture 4 ± 1cm
K^+ track radii : 110 cm ; picture 5 ± 1cm

part 2

x ——— +

picture 1

p1 : (top curve) : 55cm radii : ~~[illegible]~~ (misread) (theta 1) ~~[illegible]~~ 42° ±1° (remeasured)
p2 : (bottom curve) : 70cm radii : ~~[illegible]~~° (theta 2) 27° ± 1° [± 1° uncertainty]

θtotal : ~~[illegible]~~ (oops) 69° ± 1°

length of K^0 track : 1.6 cm ± 0.1 cm

tick marks distance : 7.4 cm ± 0.1 cm = scale : [x ——— + = 0.1 m]

B = 1.4 Tesla . 1.2 - 1.5

9/2/2021
Cassidy Bliss

Part 2 picture 2:

p1 : (top curve) : 26 ± 1 cm radii

P_2 : (bottom curve) : 115° ± 1 cm radii

p_1 : θ_1 : 59° ± 1°

P_2 : θ_2 : (remeasured) 30° ± 1°

θ total = 89° ± 1°

lenth K° track : 0.3 cm ± 0.1 cm

Scale
x → +
7 cm
± 0.1 cm

Part 2 picture 3:

p_1 : (top curve) 85 cm radii

p_2 : (botom curve) : 75 cm radii

(± 1 cm uncertainty)

p_1 : θ_1 : 34° ± 1°

p_2 : θ_2 : (remeasured) 47° ± 1°

θ total: 82 ± 1

length K° track; 1.1 cm ± 0.1 cm

Scale
x → +
7.4 cm
± 0.1 cm

Luke Z

9/21/2021
Cassidy Bliss

part 2 picture 4:

p_1: (top curve): 32 cm ± 1 cm radii

p_2: (bottom curve): 65 cm ± 1cm radii

p_1: θ_1: [illegible] remeasured 35 ± 1°

p_2: θ_2: 45° ± 1°

θ total: 80° ± 1°

K^0 length track: 0.9 cm ± 0.1 cm

Scale

x → +

7.4 cm

± 0.1cm

part 2 picture 5:

p_1: (top curve): 80 cm ± 1 cm

p_2: (bottom curve): 65 cm ± 1cm

p_1: θ_1: 36° ± 1°

p_2: θ_2: 46° ± 1°

θ total: [illegible] (remeasured) 82° ± 1°

K^0 track length: [illegible] oops wrote wrong place

0 cm ± 0.1cm

Scale

x → +

7.4 ± 0.5 cm