# PHYS 2150

# #7: Emission Spectra & the Balmer Series of Hydrogen

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## Abstract/Introduction

In this lab we derive the wavelengths of the spectral lines of the Hydrogen Balmer Series as well as obtain an experimental value for the Planck constant. We are able to do this by exploiting the modes of energy states within elemental gases, which are excited by an electrical current. The collisions that occur when the atoms are excited cause energy states to fluctuate, releasing photons of particular identifying wavelengths of the each element. The wavelengths of electromagnetic radiation that are on the visible spectrum are then directed towards a prism with a lense, so that the spectral lines can be observed through a telescope, and their angle relative to the emission measured. In this lab, we used mercury and helium to calibrate our apparatus/setup, and then calibrated the focus using lenses and telescope adjustments. Then, we were able to observe the Balmer series for Hydrogen, measuring the angles. We used a least-squared fitting procedure to fit our calibration curve to a fourth-order polynomial. We used this calibration curve and resulting representative function approximate the wavelengths of each spectral Hydrogen line. Once we have our experimental wavelengths, we can use the equation $\frac{1}{\lambda} = \left(\frac{e^2}{4\pi_o}\right)^2 \left(\frac{2\pi^2 m}{\text{ch}^3}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ to obtain h (planck’s constant). The results for experimentally derived wavelengths for the red, yellow, blue-green, and violet spectral lines of Hydrogen were: $6.34 \times 10^{-7}$, $5.92 \times 10^{-7}$, $4.79 \times 10^{-7}$, and $4.2 \times 10^{-7}$ (in meters) respectively. The percent error of these wavelengths were found to be: 3.3 %, 21.7 %, 10.3 % and 3.0 % , respectively. Our calculated uncertainty for wavelength was: $3.15 \times 10^{-9}$ which was far too small to consider our results in agreement with the actual values for wavelengths of any and all of the spectral lines of hydrogen. Our experimental values for Plank’s constant from each of the wavelengths and corresponding modes (using the equation above) were: $6.55 \times 10^{-34}$, $6.85 \times 10^{-34}$, $6.69 \times 10^{-34}$, and $6.69 \times 10^{34}$ for red, yellow, blue-green and violet, respectively. This gave us an average experimental planck constant of 6.79 x

$10^{-34}$. When compared to the accepted value of planck's constant of: `6.63 x 10`$^{-34}$, we found our percent error to be around 2.47 %. Our calculated random and systematic uncertainties were found to be: `around 9.7` × `10`$^{-36}$ and 8.1 x $10^{-17}$, respectively. We concluded that our experimental value for plank's constant did agree with the accepted value.

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# Apparatus:

The set up involved in this lab required a few different parts. One part was the enclosed mercury, helium and hydrogen gases which were integrated into a box which provided the electrical charge to be passed through the elemental gas. These three pieces were in their own enclosed unit, with the transparent container for the gases in the front, exposed, creating a lamp when the units are powered on. A free-standing lense was the second piece, which was easily moved around, but placed between the lamp and the prism table. The prism table was the third piece, which consisted of: the prism (and a way to lock it into place), a rotating table underneath attached to the telescope, so the angle at which the light is viewed could be changed, a collimator (having an adjustable slit for exposure) to absorb photons not traveling in a straight path to the prism, and the vernier and fixed scale to read the angles from. There was also a dark cloth to lay over the setup, to block out any interfering light from the room. The index of refraction of the glass is a function of wavelength, and therefore the prism refracts light of different wavelengths at different angles. This creates copies of the image of the slit (within the collimator) at every visible wavelength (color). This means that the colors we observed were in order of frequency, with higher frequencies having larger angles. Once focused properly, the lines are distinct, thin, and bright where the signature wavelengths are present.

A sketch of the setup is provided below in *figure 1*:

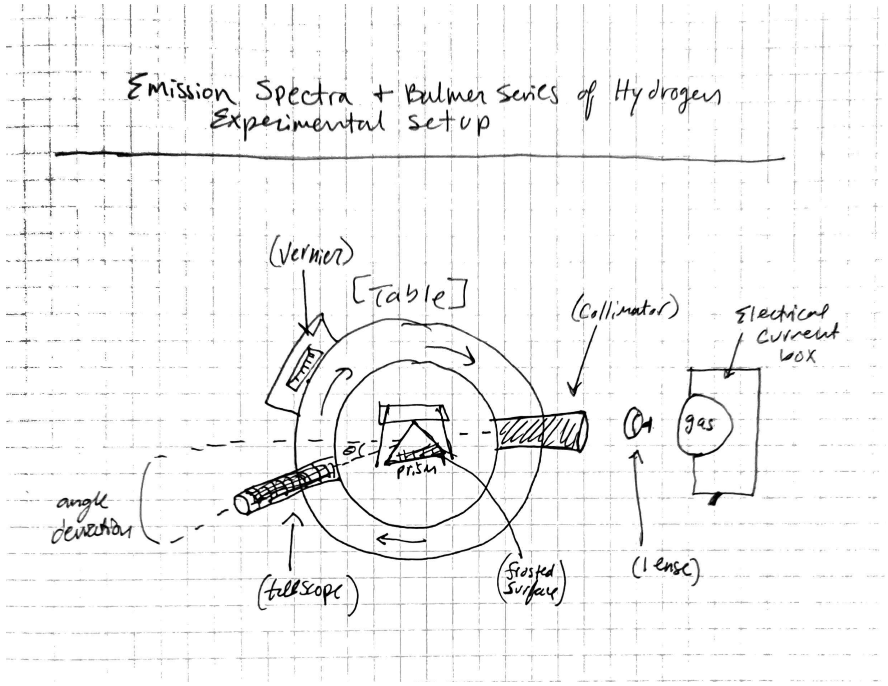

*figure 1*

# Description of Experimental Procedure

**Alignment:**
To align the apparatus, we started our setup in front of the mercury lamp. We placed the stand-alone lense in front of the lamp, and the table with the collimator facing the lense. We made sure the slit adjustment was at the largest setting (completely open). We looked through the telescope to adjust the angle until we could see the lines.

**Angle of minimum deviation:**
in order to set the prism to the angle of minimum deviation, we turned the prism until the frosted side was facing between the telescope and the light source. Looking through the telescope, with the spectral lines in view, we rotated the telescope and the prism (while keeping the lines in the picture) in the direction of parallel collimator angle until the picture appeared to strop and move in the other direction. We moved back and forth between this point until just before the picture reversed direction. Then, we locked down the prism for the rest of the experiment.

**Focus the image:**
Then we were able to focus the picture by: changing the distance of the stand alone lense from the light source, changing the length of the collimator, and adjusting the lense of the telescope slightly. We also moved the box with the prism table a few times until it was about a foot away from the source. After a lot of adjustments, we were able to see the lines very clear, and namely notice a very small gap in the Yellow line, indicating that there were two yellow spectral lines very close together. We then narrowed

the slit on the collimator.

**Match color lines and record angles for mercury and helium:**
Given a table of wavelengths, intensities, and corresponding signature color lines for both mercury and helium, we matched the lines observed, first in mercury and then for helium. As we identified/ verified these lines, we recorded the angle using the crosshairs imbedded in the telescope lense as well as the vernier. We laid tape across the table to make shifting our set up pieces easier and more accurate from the start as we moved between lamps.

**Record angles/ colors for the Balmer series of Hydrogen:**
Lastly, we were able to move our setup carefully in front of the hydrogen lamp and record the angles in the same manner as mercury and helium, noting which colors were visible and angles measurable with the crosshairs in the telescope.

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# Results

## Part 1: Mercury Data

This section contains the angle data collected for the spectral lines of Mercury in degrees We also store the known wavelengths of the spectral lines in Mercury in angstrom.

In[4]:=
```
(*Measured angles for each color in degrees*)

θHgRed  = 89 + (30 / 60) + (4 / 60); (*degrees*)
θHgYellow1  = 88 + (30 / 60) + (9 / 60); (*degrees*)
θHgYellow2  = 88 + (30 / 60) + (15 / 60); (*degrees*)
θHgGreen = 88 + (30 / 60) + (1 / 60); (*degrees*)
θHgBlueGreen = 87 + (30 / 60) + (25 / 60); (*degrees*)
θHgBlue = 86 + (30 / 60) + (17 / 60); (*degrees*)
(*first violet line was too faint to see*)
θHgViolet2 = 86 + (5 / 60); (*degrees*)

 λHgRed = 6907; (*angstroms*)
λHgYellow1 = 5791; (*angstroms*)
λHgYellow2 = 5770; (*angstroms*)
λHgGreen = 5461; (*angstroms*)
λHgBlueGreen = 4916; (*angstroms*)
λHgBlue = 4358; (*angstroms*)
λHgViolet2 = 4047; (*angstroms*)
```

---

## Part 2: Helium Data

This section contains the spectral line measurements of angles for Helium in degrees. We also store the known wavelengths of each spectral line for Helium in angstrom.

In[18]:=
```
(*Measured angles for each color in degrees*)
θHeRed1 = 89 + (30 / 60) + (13 / 60);(*degrees*)
θHeRed2 = 89 + (22 / 60);(*degrees*)
θHeYellow = 88 + (30 / 60) + (16 / 60);(*degrees*)
θHeGreen1 = 88 + (10 / 60);(*degrees*)
θHeGreen2 = 87 + (30 / 60) + (25 / 60);(*degrees*)
θHeBlueGreen = 87 + (30 / 60) + (16 / 60);(*degrees*)
θHeBlue1 = 87 + (20 / 60);(*degrees*)
θHeBlue2 = 87 + (2 / 60);(*degrees*)
(*Third blue line was too dim to see*)
θHeViolet = 86 + (30 / 60) + (21 / 60);(*degrees*)
(*second violet line was too dim to see*)


λHeRed1  = 7065; (*angstroms*)
λHeRed2  = 6678; (*angstroms*)
λHeYellow  = 5876; (*angstroms*)
λHeGreen1 = 5048; (*angstroms*)
λHeGreen2 = 5016;(*angstroms*)
λHeBlueGreen = 4922;(*angstroms*)
λHeBlue1 = 4713;(*angstroms*)
λHeBlue2 = 4471;(*angstroms*)
λHeViolet = 4026; (*angstroms*)
```

---

## Part 3: Hydrogen Data

This section contains angles measured for the spectral lines found for Hydrogen in degrees.

In[36]:=
```
(*Measured angles for each color in degrees*)
θHRed = 89 + (10 / 60); (*degrees*)
θHYellow  = 88 + (30 / 60) + (20 / 60); (*degrees*)
θHBlueGreen  =  87 + (30 / 60) + (11 / 60); (*degrees*)
θHViolet  =  86 + (30 / 60) + (9 / 60);(*degrees*)
```

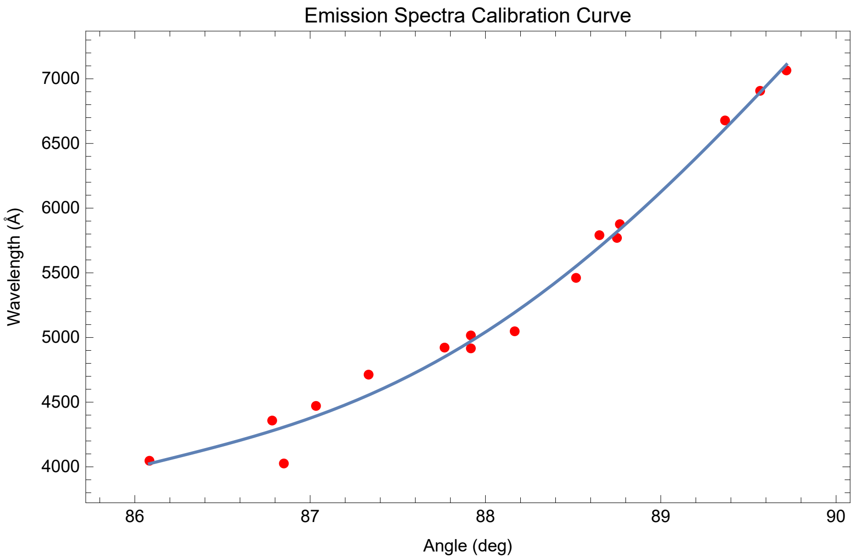
---

## Calculation: Fit Calibration Curve, Balmer Series Wavelengths, & Systematic Uncertainty

Here we store experimental angle data, and known wavelength data in their own arrays. the last array contains measured angles from Hydrogen. The least fitting squares procedure for our polynomial function/ calibration curve is implemented and a visual representation shown below. This resulted in a decent calibration curve with only a couple points notably off the resulting function. then, we were able to call the function for Hydrogen, resulting in experimental values for the wavelengths of each spectral line in the Balmer Series. The values for each are saved and converted into meters. Systematic uncertainty of $\lambda$ (wavelength) is calculated by taking the root-mean-square of the deviations in the data, resulting to be 31.51 angstrom. The formula used is as follows: $\delta\lambda = \sqrt{\frac{\texttt{Mean}[(\texttt{f}[\texttt{a}]-\texttt{b})^2]}{\texttt{Length}[\texttt{a}]-5}}$. Comparing this to the accepted values of wavelengths for each hydrogen spectral line, we found there to be a range from 3% to 21.7% error, two of which were on the lower end of the range. Therefore, our given uncertainty in meters of $3.15056\times10^{-9}$ was lower than our error values, we cannot say that our experimental values agree with the accepted value. However, we were happy that two of our lines were only 3% off from their true values.

In[44]:=
```
(*Angles measured for Hg and He*)
a = {θHgRed, θHgYellow1, θHgYellow2, θHgGreen, θHgBlueGreen, θHgBlue, θHgViolet2, θHeRed1,
   θHeRed2, θHeYellow, θHeGreen1, θHeGreen2, θHeBlueGreen, θHeBlue1, θHeBlue2, θHeViolet};
(*Known wavelengths for Hg and He*)
b = {λHgRed, λHgYellow1, λHgYellow2, λHgGreen, λHgBlueGreen, λHgBlue, λHgViolet2, λHeRed1,
   λHeRed2, λHeYellow, λHeGreen1, λHeGreen2, λHeBlueGreen, λHeBlue1, λHeBlue2, λHeViolet};
(*Angles measured for H*)
c = {θHRed, θHYellow, θHBlueGreen, θHViolet};
```

In[176]:=
```
(*Fit Calibration Curve to a fourth-order polymonial using least-squares fitting *)
d = Thread[{a, b}];
quartic = Fit[d, {1, x, x^2, x^3, x^4}, x];
Show[
 ListPlot[d, PlotRange → {MinMax[a, Scaled[.1]], MinMax[b, Scaled[.1]]}, PlotStyle → Red,
  Frame → True, Axes → False, FrameLabel → {"Angle (deg)", "Wavelength (Å)"},
  PlotLabel → "Emission Spectra Calibration Curve"], Plot[{quartic}, {x, Min[a], Max[a]}]]
f[x_] = quartic;
```



In[165]:= **(*Balmer series wavelengths in angstrom, plugging theta into fit function*)**
**f[c]**

Out[165]= {6342.34, 5916.94, 4787.47, 4224.55}

In[98]:= **(*calculated wavelenth of red line in Hydrogen:*)**
**λHRed = f[c[[1]]] *** $\left(1 * 10^{-10}\right)$ **(*meters*)**

Out[98]= $6.34234 \times 10^{-7}$

**(*calculated wavelenth of yellow line in Hydrogen:*)**
**λHYellow = f[c[[2]]] *** $\left(1 * 10^{-10}\right)$ **(*meters*)**

Out[99]= $5.91694 \times 10^{-7}$

**(*calculated wavelenth of bluegreen line in Hydrogen:*)**
**λHBlueGreen = f[c[[3]]] *** $\left(1 * 10^{-10}\right)$ **(*meters*)**

Out[100]= $4.78747 \times 10^{-7}$

**(*calculated wavelenth of violet line in Hydrogen:*)**
**λHViolet = f[c[[4]]] *** $\left(1 * 10^{-10}\right)$ **(*meters*)**

Out[101]= $4.22455 \times 10^{-7}$

**(*Systematic uncertainty using root-mean-**
**square of the devisation of data from fit line (in meters)*)**

$$\delta\lambda = \sqrt{\frac{\text{Mean}\left[(\text{f[a]} - \text{b})^2\right]}{\text{Length[a]} - 5}} * \left(1 * 10^{-10}\right) \text{ (*meters*)}$$

Out[186]= $3.15056 \times 10^{-9}$

In[180]:= **(*Actual Wavelengths of Balmer Series in m*)**
**actualHRed = 6562.10 *** $\left(1 * 10^{-10}\right)$
**actualHYellow = 4860.74 *** $\left(1 * 10^{-10}\right)$
**actualHBlueGreen = 4340.1 *** $\left(1 * 10^{-10}\right)$
**actualHViolet = 4101.2 *** $\left(1 * 10^{-10}\right)$

Out[180]= $6.5621 \times 10^{-7}$

Out[181]= $4.86074 \times 10^{-7}$

Out[182]= $4.3401 \times 10^{-7}$

Out[183]= $4.1012 \times 10^{-7}$

In[188]:= **(*Percent Error Red Line in Hydrogen*)**

$$\text{percentErrorHRed} = \frac{\text{Abs[actualHRed} - \lambda\text{HRed]}}{\text{actualHRed}} * 100 \text{ (*\%error*)}$$

Out[188]= 3.34891

```
           (*Error in value for wavelength in meters*)
           errorHRed = Abs[actualHRed - λHRed]

Out[189]=  2.19759 × 10^-8

In[191]:=  (*Percent Error Yellow Line in Hydrogen*)
                                 Abs[actualHYellow - λHYellow]
           percentErrorHYellow = ───────────────────────────── * 100 (*%error*)
                                         actualHYellow

Out[191]=  21.7292

In[192]:=  (*Error in value for wavelength in meters*)
                          Abs[actualHYellow - λHYellow]
           errorHYellow = ─────────────────────────────
                                  actualHYellow

Out[192]=  0.217292

In[193]:=  (*Percent Error BlueGreen Line in Hydrogen*)
                                    Abs[actualHBlueGreen - λHBlueGreen]
           percentErrorHBlueGreen = ─────────────────────────────────── * 100 (*%error*)
                                             actualHBlueGreen

Out[193]=  10.3079

In[194]:=  (*Error in value for wavelength in meters*)
           errorHBlueGreen = Abs[actualHBlueGreen - λHBlueGreen]

Out[194]=  4.47372 × 10^-8

In[196]:=  (*Percent Error Violet Line in Hydrogen*)
                                 Abs[actualHViolet - λHViolet]
           percentErrorHViolet = ───────────────────────────── * 100 (*%error*)
                                         actualHViolet

Out[196]=  3.00767

In[197]:=  (*Error in value for wavelength in meters*)
           errorHViolet = Abs[actualHViolet - λHViolet]

Out[197]=  1.23351 × 10^-8
```

## Calculation: Planck’s constant from each H line of Balmer Series & Random Uncertainty

Here we calculate Planck’s constant from each spectral line in Hydrogen as well as the random uncertainty in Planck’s constant. The formula used to derive the experimental Planck’s constant is as follows: $p = \sqrt[3]{\left(\frac{\lambda}{c}\right)\left(\frac{e^2}{4\,\pi\circ}\right)^2 (2\,\pi^2\,m)\left(\frac{1}{n_f} - \frac{1}{n_i}\right)}$ . The results were as follows for the red, yellow, blue-green, and violet lines, respectively: $6.55 \times 10^{-34}$, $7.07 \times 10^{-34}$, $6.85 \times 10^{-34}$, $6.69 \times 10^{-34}$ meters. The average value between these lines was found to be $6.79 \times 10^{-34}$. Comparing our average to the accepted value for Planck’s constant, we found our error to be 2.47 % error. We then calculated our uncertainty by using standard deviation of the mean as follows: $\sigma_x = \sqrt{\left(\frac{1}{4} * \left((h1 - avgh)^2\ (h2 - avgh)^2 + (h3 - avgh)^2 + (h4 - avgh)^2\right)\right)}$; $\sigma_{\overline{x}} = \frac{\sigma_x}{\sqrt{4}}$. This resulted in a random uncertainty of $9.71 \times 10^{-36}$ for planck's constant. Systematic uncertainty was calculated using quadrature summation where $\Delta h = \frac{\partial h}{\partial \lambda}(\Delta \lambda)$. Given our uncertainty, we can say that our experimental value of planck's constant agrees with the accepted value considering both systematic and random uncertainties.

In[148]:=

```
(*Known values*)
permConst = 8.85418782 * 10^-12; (*m^-3 kg^-1 s^4 A^2*)
speedOfLight = 3 * 10^8;(*meters/sec*)
emass = 9.10938356 * 10^-31 ;(*Kg*)
echarge = 1.60217662 * 10^-19; (*Coulombs*)
actualPlanck = 6.6260704 * 10^-34;

(*Formula for Planck's const in a function form*)
```

$$F[\lambda\_, nf\_, ni\_] := \left(\frac{\lambda}{speedOfLight} * \left(\frac{echarge^2}{4 * \pi * permConst}\right)^2 * \left(2 * \pi^2 * emass\right) * \left(\frac{1}{nf^2} - \frac{1}{ni^2}\right)\right) \wedge (1/3)$$

In[154]:=

```
(*Experimental value for Planck's constant Red Hydrogen Line*)
h1 = F[λHRed, 2, 3]
```

Out[154]= $6.55008 \times 10^{-34}$

In[155]:=

```
(*Experimental value for Planck's constant Red Hydrogen Line*)
h2 = F[λHYellow, 2, 4]
```

Out[155]= $7.07359 \times 10^{-34}$

In[156]:=

```
(*Experimental value for Planck's constant Red Hydrogen Line*)
h3 = F[λHBlueGreen, 2, 5]
```

Out[156]= $6.84514 \times 10^{-34}$

In[157]:=

```
(*Experimental value for Planck's constant Red Hydrogen Line*)
h4 = F[λHViolet, 2, 6]
```

Out[157]= $6.69057 \times 10^{-34}$

In[158]:=

```
(*Average Plank Value*)
```

$$avgh = \left(\frac{h1 + h2 + h3 + h4}{4}\right)$$

Out[158]= $6.78985 \times 10^{-34}$

```
(*Error Planck's Constant experimental vs actual*)
```

$$\text{percentErrorP} = \frac{\text{Abs}[\text{avgh} - \text{actualPlanck}]}{\text{actualPlanck}} * 100$$

Out[164]= 2.47168

In[187]:= `ErrorP = Abs[avgh - actualPlanck]`

Out[187]= $1.63775 \times 10^{-35}$

In[160]:= `(*Standard Deviation*)`

$$\sigma_x = \sqrt{\left(\frac{1}{4} * \left((\text{h1} - \text{avgh})^2 + (\text{h2} - \text{avgh})^2 + (\text{h3} - \text{avgh})^2 + (\text{h4} - \text{avgh})^2\right)\right)}$$

Out[160]= $1.94239 \times 10^{-35}$

```
(*Standard Deviation of the mean ~Random Uncertainty of planck's constant*)
```

In[161]:= $\sigma_{\overline{x}} = \frac{\sigma_x}{\sqrt{4}}$

Out[161]= $9.71194 \times 10^{-36}$

In[162]:= $\delta h = \sigma_{\overline{x}}$

Out[162]= $9.71194 \times 10^{-36}$

In[198]:= `(*Systematic Error in planck's constant*)`

$$\Delta h = \sqrt{\left(\left(\text{Abs}\left[\frac{\delta h}{\delta \lambda}\right] * \lambda\text{HRed}\right) + \left(\text{Abs}\left[\frac{\delta h}{\delta \lambda}\right] * \lambda\text{HYellow}\right) + \left(\text{Abs}\left[\frac{\delta h}{\delta \lambda}\right] * \lambda\text{HBlueGreen}\right) + \left(\text{Abs}\left[\frac{\delta h}{\delta \lambda}\right] * \lambda\text{HViolet}\right)\right)}$$

Out[198]= $8.0976 \times 10^{-17}$

---

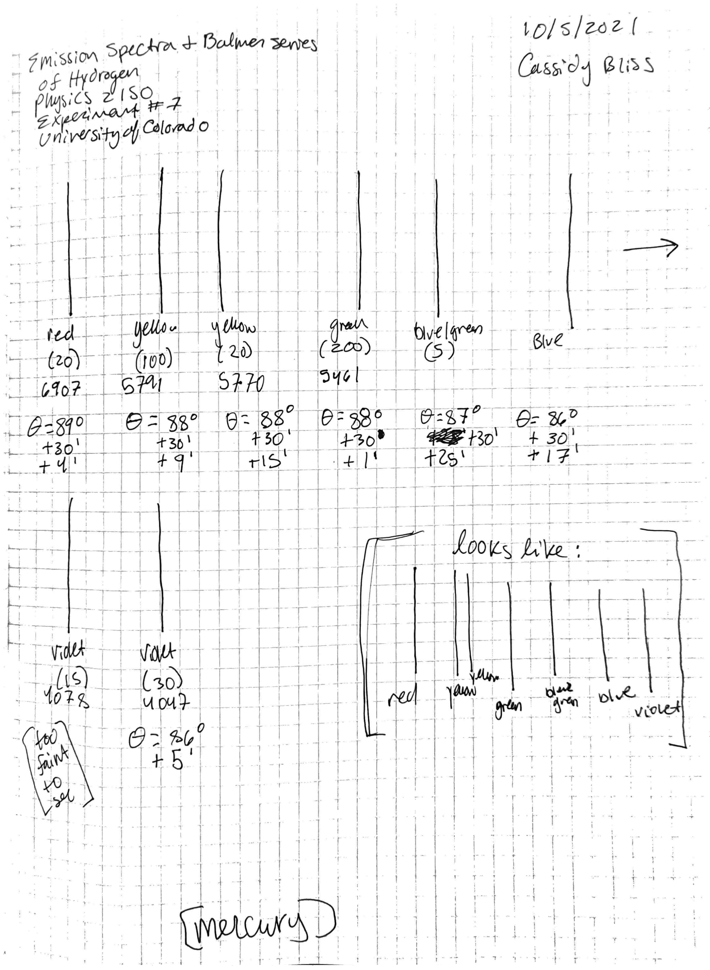
# Discussion

Overall, we conclude that you can successfully derive Planck’s constant within our uncertainty using the Balmer series spectral lines of hydrogen after proper calibration using mercury and helium. Our experimental values for the wavelengths of each line in hydrogen were not close enough, given our systematic uncertainty to agree with accepted values. The error found for the red, yellow, blue-green, and violet lines were around 3.3%, 21.7%, 10.3%, and 3.0%, respectively. Our uncertainty for wavelength was calculated to be around $3.15 \times 10^{-9}$ meters, which was too low for our errors. However, they were close enough to derive an experimental value for planck’s constant in the second part of our analysis. Our average experimental value for planck’s constant was $6.78985 \times 10^{-34}$, and when compared to the accepted value of $6.6260704 * 10^{-34}$, we found a 2.4% error. Our random uncertainty of $9.71194 \times 10^{-36}$ and systematic uncertainty of $8.0976 \times 10^{-17}$ concluded that our experimental value did agree with the accepted value. Sources of error may have stemmed from several places within our experiment. First, in angle of minimum deviation, it was very difficult to place the prism exactly before the image appeared to stop and begin reversing direction, and some displacement in angles could have resulted. Second, the crosshairs within the telescope were very difficult to see, namely on the fainter lines. Therefore, the precise angle at which each line was refracted could have been off by an amount of minutes, which should have been included in our uncertainty, and if done so, could have resulted in our results agreeing with accepted values. Thirdly, measurements taken from the vernier may have been read improperly as this was our first time working with such a measurement device, and seeing the lines, namely for the minutes was difficult for both my lab partner and I given we have poor vision.

# Conclusion

In conclusion, using prism and elemental gas spectroscopy, we were not able to derive wavelengths for the balmer series of hydrogen that could be said to agree with the accepted values, based on the uncertainty calculated. However, our experimentally derived wavelengths for the hydrogen spectral lines were within our uncertainty when we calculated our experimental Planck’s constant. Overall we consider this lab a success.

# Scanned Sheets from Lab Notebook



Emission Spectra + Balmer series
of Hydrogen
Physics 2150
Experiment #7
University of Colorado

10/5/2021
Cassidy Bliss

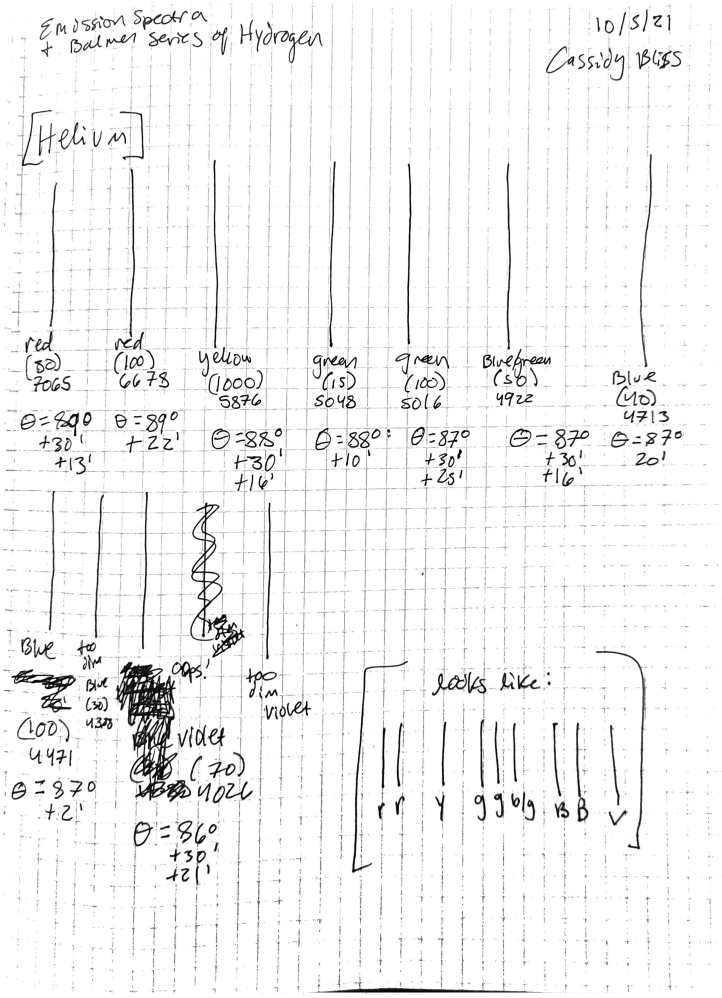
|  |  |  |  |  |  |
|---|---|---|---|---|---|
| red | yellow | yellow | green | blue/green | Blue |
| (20) | (100) | (20) | (200) | (5) |  |
| 6907 | 5791 | 5770 | 5461 |  |  |
| θ = 89° | θ = 88° | θ = 88° | θ = 88° | θ = 87° | θ = 86° |
| +30' | +30' | +30' | +30' | +30' | +30' |
| +4' | +9' | +15' | +1' | +25' | +17' |

→

|  |  |
|---|---|
| violet | violet |
| (15) | (30) |
| 4078 | 4047 |
| too faint to see | θ = 86° |
|  | +5' |

looks like:

[mercury]



Emission Spectra
+ Balmer Series of Hydrogen

10/5/21
Cassidy Bliss

[Helium]

red
(80)
7065
θ = 89°
+30'
+13'

red
(100)
6678
θ = 89°
+22'

yellow
(1000)
5876
θ = 88°
+30'
+16'

green
(15)
5048
θ = 88°
+10'

green
(100)
5016
θ = 87°
+30'
+25'

Bluegreen
(50)
4922
θ = 87°
+30'
+16'

Blue
(40)
4713
θ = 87°
20'

Blue
(100)
4471
θ = 87°
+2'

too
dim
Blue
(30)
4388

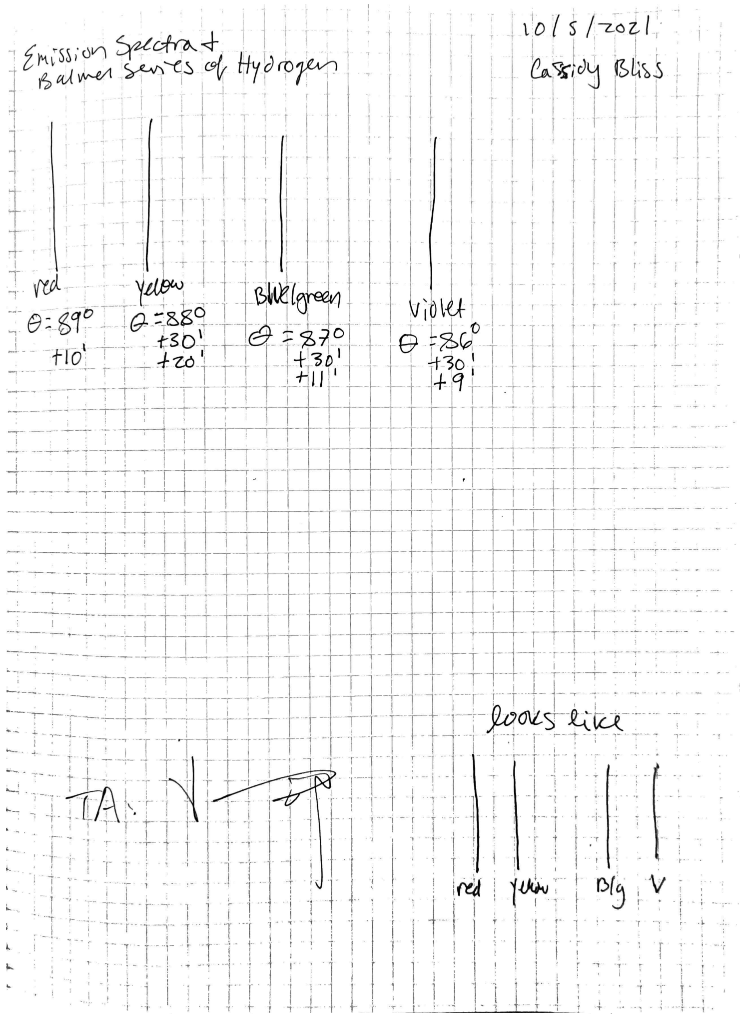
Oops!

violet
(70)
4026
θ = 86°
+30'
+21'

too
dim
violet

looks like:

r r Y g g b/g B B V



Emission Spectra + Balmer Series of Hydrogen

10/5/2021

Cassidy Bliss

red

$\theta = 89^\circ$ +10'

yellow

$\theta = 88^\circ$ +30' +20'

Blue/green

$\theta = 87^\circ$ +30' +11'

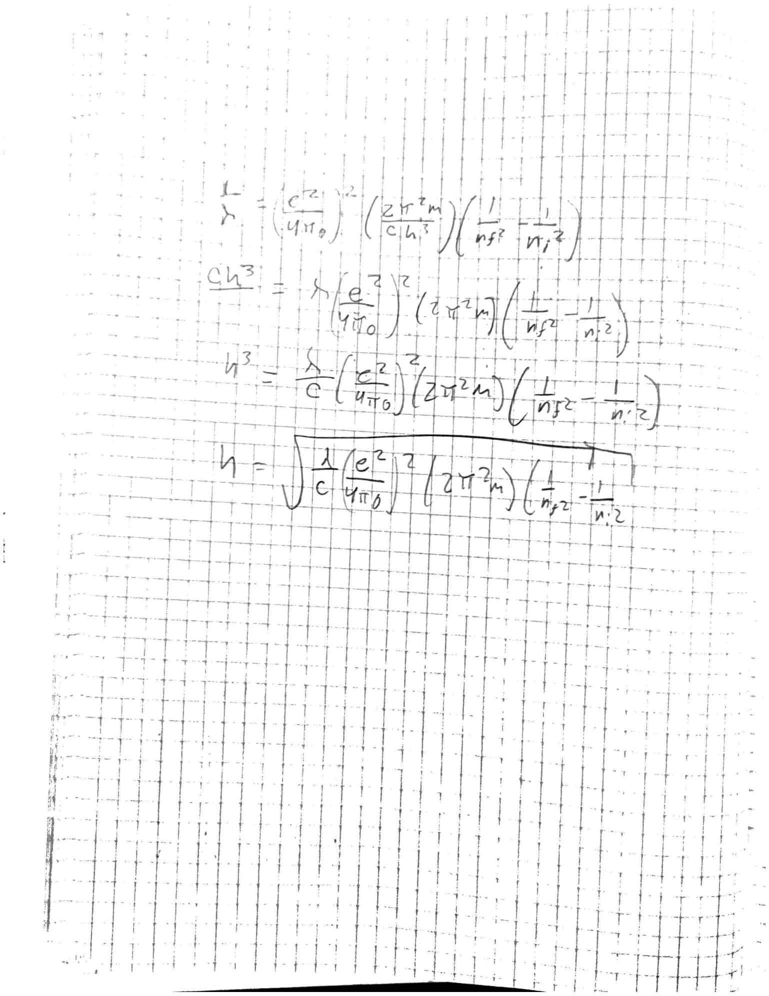
violet

$\theta = 86^\circ$ +30' +9'

looks like

red yellow B/g V

TA:



$$\frac{1}{\lambda} = \left(\frac{e^2}{4\pi_0}\right)^2 \left(\frac{2\pi^2 m}{c h^3}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\underline{ch^3} = \lambda \left(\frac{e^2}{4\pi_0}\right)^2 (2\pi^2 m)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$h^3 = \frac{\lambda}{c}\left(\frac{e^2}{4\pi_0}\right)^2 (2\pi^2 m)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$h = \sqrt{\frac{\lambda}{c}\left(\frac{e^2}{4\pi_0}\right)^2 (2\pi^2 m)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$